Sustainable strategies for knowledge of built heritage: graphic methods for vaulted and arched masonry structures

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Abstract

Traditional materials and construction techniques are of central importance for knowledge and, subsequently, the protection of existing building heritage, including historical and monumental sites. The evaluation of the static consistency of existing structures can be carried out in a coherent and accurate manner only by starting from an in-depth knowledge of the geometric, physical and mechanical characteristics of the building's structural elements. In recent years, we have witnessed an exponential increase in the use of advanced technologies for graphic representation and subsequent structural analysis, especially in the study of existing buildings. For a real and concrete application of the results, these representations must be critical and functional for understanding the building rather than merely serving as graphic virtuosity. In this framework, the graphic methodologies to be applied, in support of digital data, in simplified procedures for a preliminary knowledge of the buildings, their technological configuration and static behaviors have been studied in depth. It is important to point out that the historic built heritage, unlike the modern built heritage, is characterized by the strong presence of vaulted and arched elements, with columns and piers of various shapes and materials; for this reason, models and procedures of graphic analysis based on the limit analysis and the theories of Jacques Heyman have been applied. The method is first applied and verified on a single historical monumental building to test its limits and potential and then applied to other case studies. The results show that the graphical and analytical analysis of structures is a valid and reliable tool for analyzing buildings in order to understand their structural behavior, since, even if the model is simplified, it is possible to obtain results that are strongly correlated with the behaviors of the structures and can guarantee good accuracy and adequate safety margins.

Keywords: Built heritage, Structural analysis, Graphic method, Morocco, Construction techniques.

1. Introduction

The culture of the valorization and preservation of Cultural Heritage, or in general of building recovery, has always recognized the "built environment" as the bearer of a quantity of Values that are defined but changeable according to the historical epoch with respect to this, it is necessary to define decisions on the possibilities of intervention in order to respond to the continuous and constant need for maintenance and also transformation of this heritage [1]. In recovery, reuse and refurbishing projects, the impact of modernity can also take on destructive aspects. There are several studies from which emerge the need to address recovery, refunctionalisation and infrastructural integration, through an understanding of the urban environment and the opportunity to define a cultural guideline, a sort of code of behavior that allows for the re-inhabitation of this architectural heritage. Bringing the structures back to life in accordance with

49   the requirements of modern living, without altering their consolidated characteristics over time, with interventions
50   correlated to the original constructive, typological, functional and technological characteristics, is indispensable, but at
51   the same time, particular attention must be paid to the integration and sustainability of the new interventions [2]. In
52   this cultural context, the analysis of existing structures, in particular historical and monumental buildings, in general
53   implies confrontation with masonry constructions, made of a material in which a resistant structure is often not clearly
54   recognizable. This depends on the building's geometry, the distribution of stiffnesses and masses within it, the temporal
55   succession of construction works and subsequent modifications, the urging actions, and sometimes the presence of
56   structural cracks and material damages, which may be more or less evident [3]. In these fields, the study and numerical
57   modeling of the structure, also of the F.E.M. type, cannot be separated from analysis tools that allow us to describe it
58   through a first decomposition and subsequent independent schematization of single parts of the construction (arches,
59   vaults, piers, walls, etc.), otherwise the instrument of critical control of the results would be missing. The analysis of
60   the static condition of existing structures and historic and monumental buildings can therefore only be conducted
61   consistently and correctly based on a thorough knowledge of the geometric, physical and mechanical characteristics of
62   the building's structural elements [4]. The latter consists of building materials and techniques that vary considerably in
63   relation to both the geographical area and the era of construction, as well as the various historical stratifications.
64   Consider also the difficulties in representing and managing the large and complex knowledge related to the non-
65   geometrical aspects of the historical heritage with the problems related to meeting the requirements of semantic
66   representation of the built heritage [5]. For this reason, they must be evaluated on a case-by-case basis with a careful
67   examination of the building in its entirety through geometric measurements, surveys, and non-destructive testing [6].
68   Researching, investigating and discovering what the orders and rules of good building are, is a subject that has been
69   settled in the debate of the scientific community, but it proves to be more topical than ever if we consider, in Italy alone,
70   the wide diffusion and peculiarity of building techniques, autochthonous materials, specificity of workers, unique to
71   urban contexts that are never standardized, such as historic centers. In such diversity, difficult to standardize, the
72   meaning of intervening "according to the rules of the art" acquires the meaning of "doing with care and precision",
73   following dimensional, constructive and formal language rules dictated in some cases also by a component of
74   indeterminacy, the result exclusively of man's need for survival [7]. Within this debate, therefore, an attempt is made
75   to establish the definition of a "methodological strategy" that can connect the recovery intervention essentially aimed
76   at the reinterpretation of pre-existing structures, with a reading of historical forms and materials. The pivot around
77   which the entire discussion revolves is the definition of a method for transferring knowledge from the past to meet the
78   demands of the present, while considering future demands and needs. A suitable methodological approach, therefore,
79   is the key to ensuring the processual continuity of the built heritage. This approach allows us to interpret and read the
80   existing building heritage in the light of contemporary needs; millennial history must confront the demands of the
81   current lifestyle. It is therefore important to continue to implement and develop different methodological approaches
82   that propose, in addition to preserving the image of pure matter, innovative formal and technological solutions starting
83   from the basics of static knowledge of masonry constructions, tackling, in a simple and innovative way, the study of
84   the behaviour of both the most important monuments and the most common works, in a unitary vision that crosses
85   engineering and architecture [8]. In this perspective, knowledge of the artifact as an object of analysis must be obtained
86   by exploiting all the most modern and advanced technologies, such as the use of drones, 3D laser scanners, etc., aimed
87   at the most complete and comprehensive graphic rendering possible. However, at the same time, such representations
88   must be both critical and functional to the understanding of the artifact, rather than merely a display of graphic
89   virtuosity. From another perspective, the widespread use of finite element models, facilitated by the characteristics of
90   increasingly prevalent IT tools, requires particular attention and care. The modeling of existing buildings, and of those
91   in masonry with historical and monumental interest, is indeed very complex, especially in relation to the possible
92   schematizations for the realization of the model and the choice of the mechanical characteristics of the materials. Based
93   on geometric and dimensional measurements, even if carried out with high accuracy, it is of fundamental importance
94   to thoroughly investigate the building and increase its knowledge, both in order to create a mathematical model
95   congruent to the behavior of the structures, to correctly interpret it, and to verify the results. For each structural element,
96   the geometry and nature of the materials must be known, because only this can allow obtaining a first configuration
97   that is essential for a full understanding of the overall static functioning of the building. This process of first approach,
98   so indispensable for the real and complete knowledge of the artifact, is effectively realized through analysis using
99   graphic methods, based on the limit analysis and the theories of Jacques Heyman [9-10]. As further study, if necessary,
100 through subsequent steps, it is then possible to arrive at the finite element (F.E.M.) modeling of both the individual
101 elements and the whole structure, in a critical way, ensuring a check of the input and output data, to reduce the risk of
102 incorrect interpretations of the results of the mathematical model. To test the potential and limits of the proposed graphic
103 method, it was applied in advance to a case study that was well-suited to the purpose. A historical building in ordinary,
104 regular and compact load-bearing masonry was analyzed, which presented typological, constructive and structural

105   elements, typical of historical buildings such as arched structures, i.e. vaults and arches, piers, metal chains, etc. The
106   chosen artifact also proved to be very suitable both for the applications of Heyman's theories and for detailed and
107   meticulous FEM finite element modeling. The choice went to the "Logge di Banchi" in Pisa, Italy. The graphic method,
108   once validated through the case study, was then applied as part of a broader, international, multidisciplinary research
109   project to the monumental gate called "Bab Agnaou" of the historic walls of the city of Marrakech in Morocco.

2. Methods

111   The proposed approach for understanding and analysis of masonry structures with vaults and arched structures
112   consists of the following steps. Starting from the basis of global and local geometric measurements, the buildings must
113   be studied in depth to increase their buildings knowledge. For each structural element, the geometry, nature of the
114   materials and the constructive technique must be studied. In particular, the constructional logic of vertical structures
115   (walls and masonry fixtures), horizontal pusher structures (arches and vaults in masonry) and non-pushing (wooden
116   roof slabs) must be studied. Based on these analyses, the first framework for understanding the overall static functioning
117   is possible to obtain. Moreover, it's possible to identify the structural elements to be analyzed in detail successively. In
118   this phase of knowledge about the building, the presence of structural damage and degradation phenomena could be
119   investigated. The analysis of the structural elements is carried out using graphic methods based on limit analysis and
120   on the theories of Jacques Heyman. To conduct the graphic analysis, the structure has be decomposed by identifying
121   its significant modules. It's important to highlight that, in line with Heyman's hypothesis, the path of analysis of the
122   structure proposed is based on a model with non-reactive tensile masonry. The masonry has a tensile strength that is
123   not considered in this type of modeling. A simplification is therefore carried out, which, however, is in favor of safety.
124   It is considered an ultimate limit state, but in reality, considerable resources of the structure given by the real tensile
125   strength of the masonry are neglected. Moreover, more specifically regarding the analysis of masonry arches, it is useful
126   to underline that, in addition to considering them free of tensile strength, we consider them as free in space. In reality,
127   they are constrained in the vertical plane by the masonry, which prevents changes in shape and, therefore, the onset of
128   instability phenomena, which are the main causes of the collapse of arched and vaulted structures. This allows the arch
129   to have almost infinite compressive strength, limited only by the crushing resistance of the material, as seen in Heyman's
130   hypothesis [11-12-13]. As far as the vaulted structures are concerned, even if in a minor way, they are limited in the
131   shape change from the backward, which, if it is of good workmanship and of suitable materials, is very effective.
132   Moreover, in the analysis of the vault, carried out in sectors, the contribution that each sector offers to the other and the
133   global contribution that their union offers is neglected. From the above, it is clear that the results obtained from the
134   previous analyzes are useful for understanding the behavior of the structural elements, operating strongly in favor of
135   safety [14].

136   *2.1 Analysis of structures from graphic-synthetic to F.E.M. methods: case study "Logge di Banchi" in Pisa*

137   The complex of the "Logge di Banchi" in Pisa (Figures 1-2) is placed at the center of the city, on the historical
138   commercial axis where the palaces of political and administrative life are located. The current configuration of the
139   "Logge di Banchi" is the result of the succession of various transformative interventions that occurred during the 400
140   years of the building's life. It consists of three main parts: (1) the ground floor consisting of the Lodges built in 1603,
141   presumably based on a design by Bernardo Buontalenti, (2) the first floor partly made up of the original 17th-century
142   first floor, which was later transformed and raised in 1865 by the architects Cervelli and Piccoli called "State Archives"
143   and (3) the basement floor built in the early twentieth century and named "Albergo Diurno Cobianchi". The ground
144   floor is preserved intact in the structures and architectural elements that characterize it; the plan is rectangular,
145   measuring approximately 33 m x 19 m, with a height of 22 m [15].

146

147   Figure 1 – "Logge di Banchi" in Pisa. Façade view and internal view of the loggia.

148   Figure 2 – Technical Drawings of "Logge di Banchi". Front and back elevation - Cross and longitudinal section.

149   *2.2 Analysis of structures with graphic-synthetic methods*

150   Each internal module (Figure 3), enclosed between the four separately tested stone reinforcement arches, formed by
151   a lunette barrel vault, has been divided into 6 equidistant parts with a width of about 45 cm. Due to this schematization,
152   each lunette is divided into six sectors, and the barrel vaults are divided into 12. For each strip, the graphic analysis was
153   carried out by obtaining a funicular polygon of forces such as to be as close as possible to the arch axis line and, in any
154   case, contained within the arch. Also, for stone and masonry arches, a graphic and analytical analysis was carried out
155   for the determination of the stress states. Piers constituting the lodges can be classified, according to their position and
156   geometric structure in plan, into three main types: "L", "I", and "T". For each type of pier, the rays and the relative
157   central ellipse of inertia and the central inertial core were determined through graphic and analytical methods. For each
158   strip, the graphic analysis was carried out by obtaining a funicular polygon of forces such as to be as close as possible
159   to the arch axis line and in any case contained within the arch. (Figure 4).

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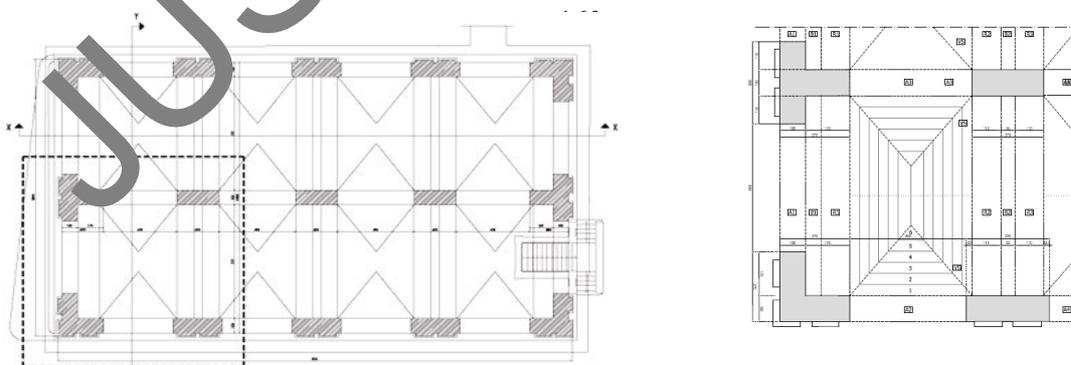
172   Figure 3 – Plan of the "significant module"
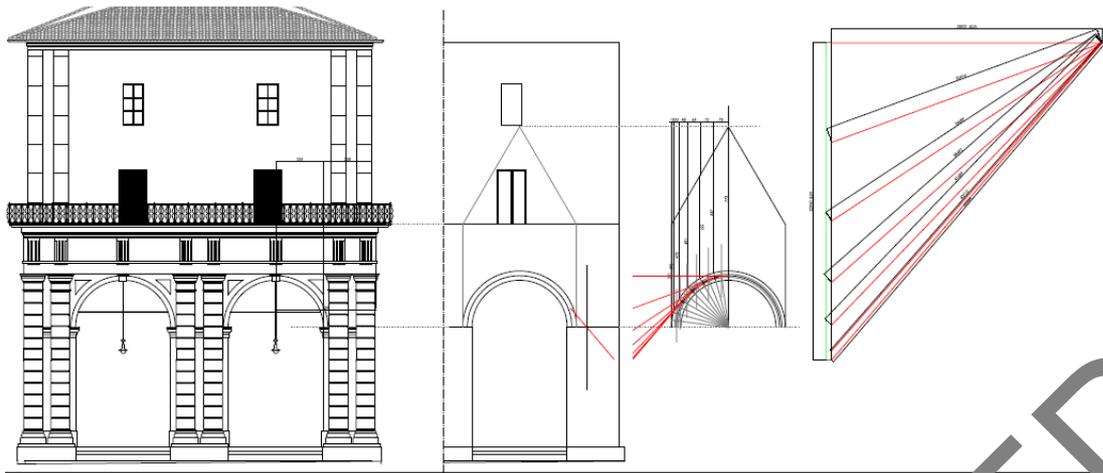
Figure 4 – Search for the "A2" arch pressure line.

2.3 F.E.M. analysis of partial and global models

Based on the analysis of the structures with synthetic graphical methods, then the creation of a model F.E.M. of the same structures has been realized. The stress and deformation states of the element have been investigated in relation to the hypotheses made, through various schematizations of load application, on-board constraints, and types of F.E.M. elements used for modeling the different types of structural elements. The numerical analysis of the structures was performed with the "Straus 7" [16]. The construction of the finite element model has been realized, without the aid of "auto mesh functions", directly constructing every structural element (arches, vaults, springers, walls, etc.) based on geometric measurements, observations and considerations made in situ. In the model, only the above-ground structures were represented, leaving out the foundation structures but schematizing their constraints (Figure 5).
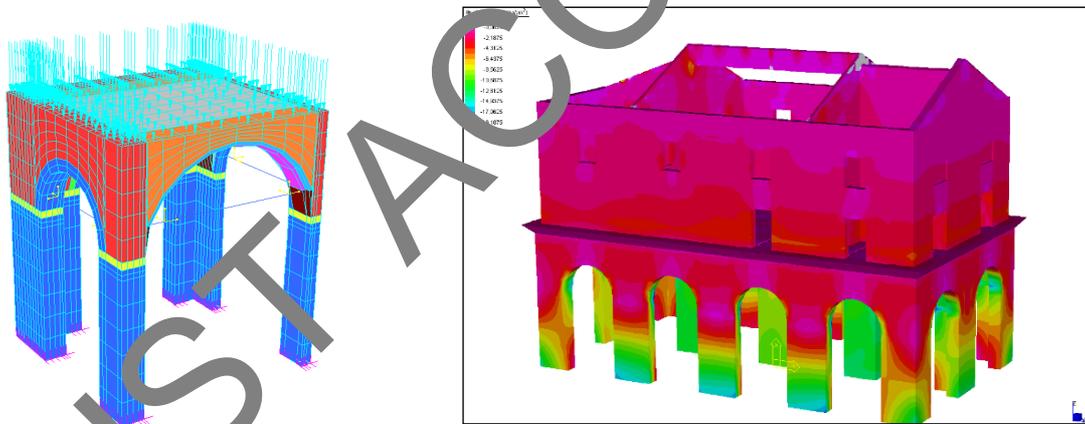
Figure 5 – Exemplary representation of the F.E.M. model of the "local significant module" (axonometric view of structural elements, loads and nodes) and "global model" (axonometric view of the structural analysis results - ZZ stresses).

2.4 Case study "Bab Agnaou in Marrakech" (Morocco)

The simplified graphic method, studied, analyzed and validated with FEM analysis in the case study of the " Loggia dei Banchi" in Pisa, has been applied, at an international level, to the Bab Agnaou, a monumental gate that is part of the historic walls of the ancient Medina of Marrakech (UNESCO site), Morocco. The study presented here is a synthesis of a much broader research carried out within the framework of an agreement between Department of Energy, Systems, Land and Construction Engineering of the University of Pisa (D.E.S.T.eC) and École Nationale d'Architecture de Marrakech (ENAM) signed since 2018 to develop joint studies in the field of Architecture, Urban Space and Technological Development. Bab Agnaou (Figures 6-7) is one of the best-known monumental gates in Marrakech and its construction is attributed to the Almohad caliph Abu Yusuf Ya'qub al-Mansur and was completed around 1188 and 1190. The gate was the main public entrance to the Royal Kasbah (citadel) in the southern part of Medina. The function

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198 of the gate was primarily decorative, given its location already inside the city walls and was originally flanked by two
199 bastion towers crowned with merlons [17-18-19].
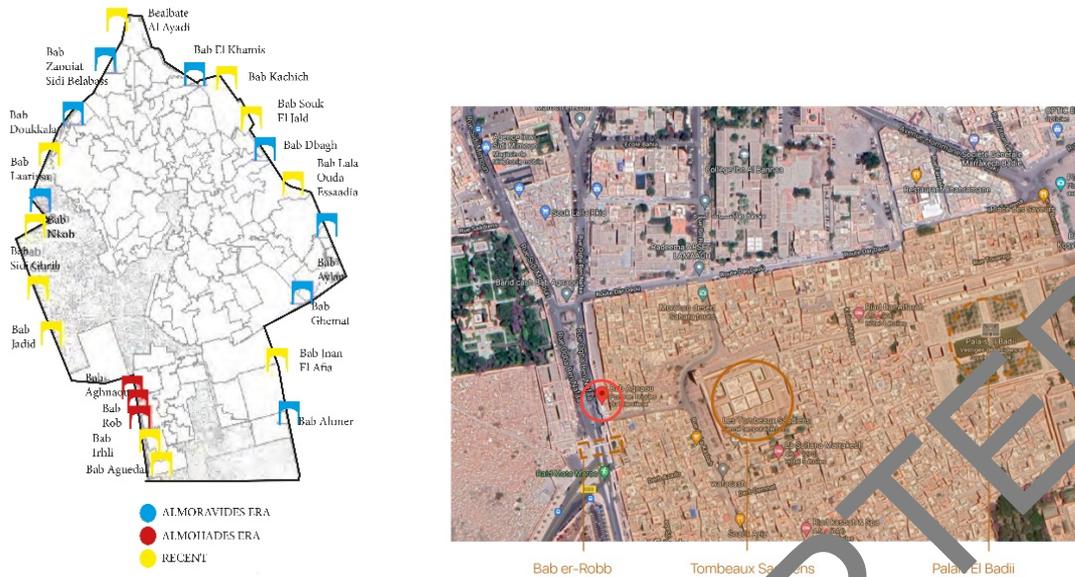
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201 Figure 6 – Location of Bab Agnaou and historical periods of the gates on the left, and other places of interest on the right.

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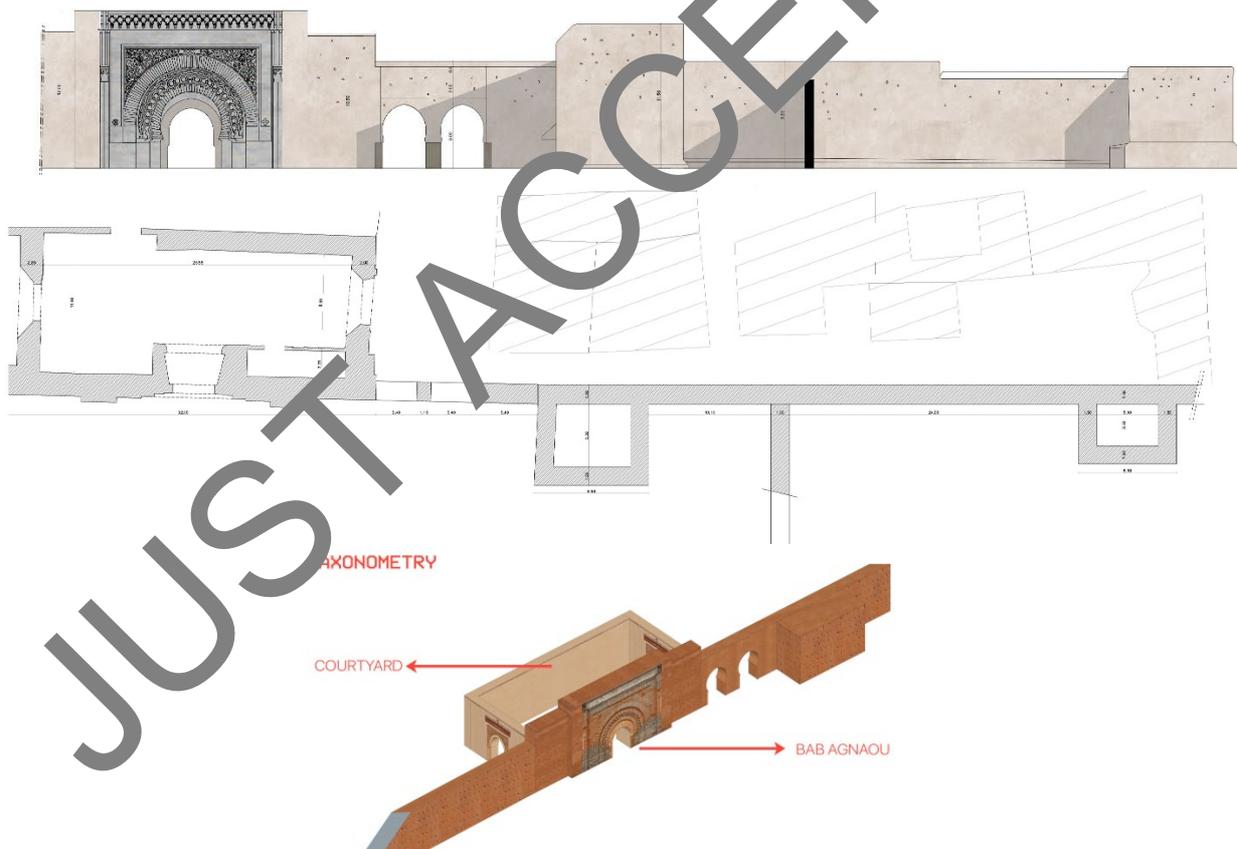
204
205 Figure 7 – Façade, plant and the axonometry view of Bab Agnaou gate.

206 The passage inside was a bent entrance passing through a large, vaulted vestibule. The flanking towers and the
207 covered vestibule, however, have since disappeared, and the archway of the gate has been partly filled in with a smaller
208 and simpler brick arch. Since its construction in the 12th century, the gate seems to have undergone fairly frequent
209 restorations (Figure 8), three of which are archived: a) the restoration of the eighteenth century, during the reign of
210 Sultan Sidi Mohammed Ben Abdellah (1757-1790) in which the arch of the gate opening was reduced in width and

211 height; b) the restoration in 1930 during the French protectorate; c) the restoration in 1960 which only proof's left are
212 photos and testimonies of former craftsmen [18].
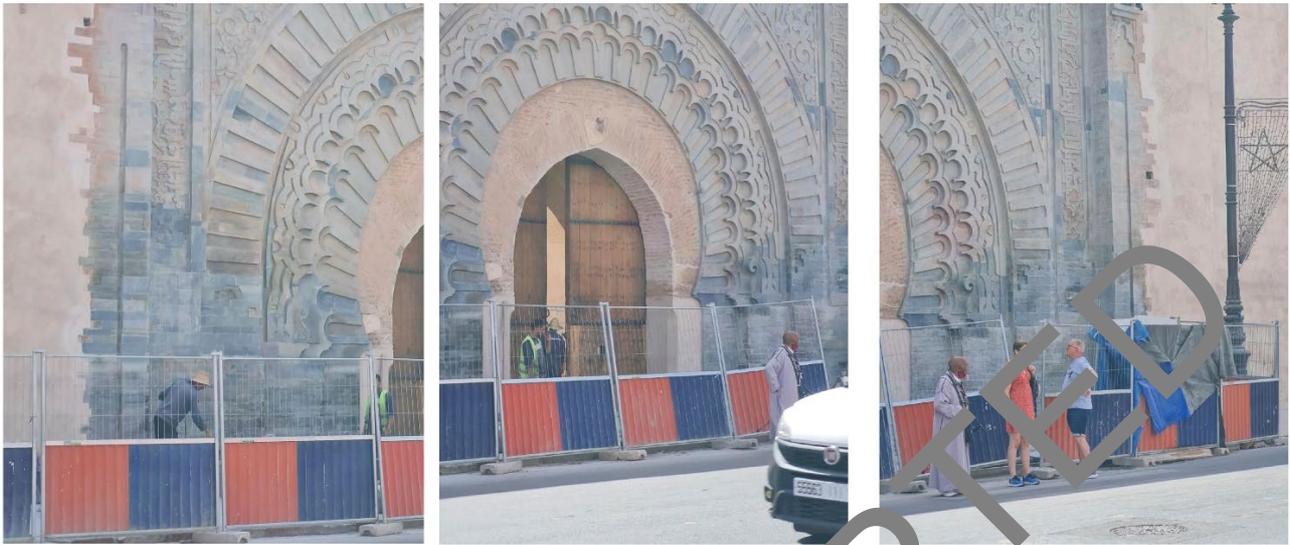
213
214 Figure 8 – Bab Agnaou gate after the end of the renovation.

Materials :

Patterns :

215

Figure 9 – Analysis of the construction materials and decorations, top. Degradation analysis map, bottom.

Bab Agnaou, with its sumptuous stone decoration, consists of 4 successive semicircular arches, which appear to be superimposed on each other (Figure 9). It is also very common in mosques to have a purely oriental arch in which the two semi-arches extend downwards below the plane of the centres. The analysis procedure proposed for the case study of the "Logge di Banchi" in Pisa was also applied to the arch in question. The load-bearing brick arch was analysed in detail, hypothesising it, for the sake of safety, as a support for the structures above. Operationally, for the graphic analysis, the arch was schematised as a round arch, radius 1500 mm, and subsequently divided into 30 ashlars, with 5 mm joints (Figure 10).
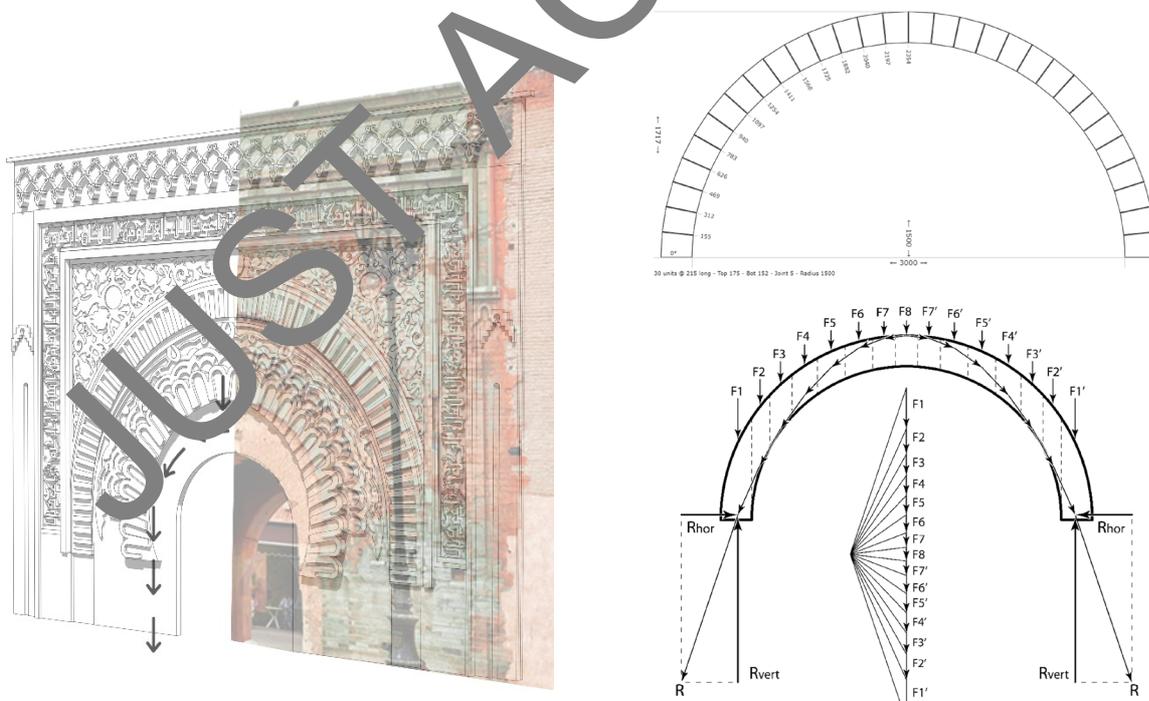

Figure 10 – Analysis of the arch of Bab Agnaou gate with the graphic method

240 ## 3. Results

241   The results obtained in the research can be divided into two parts, one for each of the two case studies analyzed. In
242 detail, we can make a first, more theoretical and in-depth consideration on the Pisa case regarding the proposed
243 simplified method, and a second, more practical and expeditious one regarding the possible applicability in a more
244 operational field. With reference to the case study of the "Logge di Banchi" in Pisa, it is possible to observe that,
245 comparing both the qualitative and the numerical results obtained both with the graphical and analytical analyses and
246 with the F.E.M. analyses, we note good correspondence, even if with slight differences due to the schematizations
247 carried out in both types of analysis. The main source of the differences in the results of the two types of analysis is
248 the schematization of the constitutive bond of the masonry material; in graphical analysis, masonry is schematized as
249 a non-resistant linear elastic material, and in the finite element analysis, the material is schematized as linear elastic but
250 also endowed with tensile strength. These results represent the two extremes or the lower and upper limits within which
251 will certainly be understood the real behavior of the masonry, of the structural elements, and of the global structure,
252 being the masonry material endowed with one, even if minimal, tensile strength.
253   Analyzing these results in detail, however, some singularities can be fully justified based on the aforementioned
254 premises; in particular, a difference in the stress state of the chains is observed, higher in the F.E.M. analysis compared
255 to the graphical analysis. This is presumably due to the different stiffness of the structures and the contributions taken
256 into consideration, or better depends on the fact that in the graphic analysis the acting forces are determined by
257 analyzing the vaulted structures by sectors, separated from each other, the separated arcs, the walls applied as separate
258 loads and then added together but without taking into consideration the mechanical loads that are actually present
259 between them. The F.E.M. model also takes into account this interaction between structural elements and their portions,
260 although considering the material as having tensile strength, and therefore slightly modifying the results. Moreover, the
261 spandrel of the vaults is, in the first case, considered as a load on the structure, while in the second case, although
262 equipped with different stiffnesses to limit the effects, it is an integral part of the structures and therefore considered
263 both as a weight and a structure interacting with the vaults. In conclusion, we can consider the global model faithful to
264 the expected behavior of the structure only if it is interpreted in light of the previous considerations.
265   Regarding the findings from the case study of Bab Agnau in Morocco, however, we can say that the graphic method,
266 with its potential and limits as tested and highlighted in the case study in Pisa, is easy to apply when inserted within a
267 study of artifacts. By applying traditional cognitive methods, historical analysis, geometric survey, material analysis
268 and degradation, etc., the graphic analysis, rapid and expeditious, constitutes a further useful element of knowledge
269 that provides elements both in terms of seismic vulnerability and the degree of static safety of the structures. From this
270 enriched cognitive framework, useful elements can emerge even more completely for the planning of possible future
271 interventions on the artifact, both for reuse and refunctionalisation and for management and scheduled maintenance.

272 ## 4. Conclusions

273   On the basis of the above considerations, it is possible to draw some general conclusions regarding the methods of
274 investigation and analysis used in this case study. The analysis of the static consistency and of the eventual evaluation
275 of the seismic vulnerability of the existing structures, in particular of the historical and monumental buildings, can be
276 carried out in a coherent and correct way only starting from a deep knowledge of the history and of the geometric,
277 physical and mechanical characteristics of the structural elements. To obtain this, a preliminary and in-depth knowledge
278 of the building in its entirety is necessary, through historical documentation, geometric measurements, investigations,
279 tests, etc.
280   Being able to read the supporting elements, making their schematization and the consequent modeling is often very
281 complex and challenging to interpret. On the contrary, through the use of graphic-synthetic and analytical methods
282 based on the method of limit analysis and the theories of Jacques Heyman, it is possible to obtain a first detailed picture
283 of the building. It allows access to understanding the functioning of structures, starting from the main elements such as
284 arches, vaults, springers, and walls, to gain a comprehensive understanding of the structure as a whole. Therefore, the
285 use of graphical and analytical analysis is not only an excellent tool for assessing the static consistency of structures,
286 with the advantages and limitations illustrated above, but also and above all an excellent tool for the knowledge of
287 monumental historical building artifacts, even in the case of complex constructions. In a second phase, through a finite
288 element analysis, everything obtained through graphic and analytical procedures is analyzed to investigate local
289 phenomena and behaviors that are difficult to analyze using only a global model. Starting with considerations and
290 evaluations based on rigorous graphical and analytical analysis of the structures, F.E.M. with particular levels of detail
291 can be realized, aimed at local investigations on even complex buildings. It is objectively difficult in structures such as
292 the structure under study to analyze all global and local aspects with a single model F.E.M. being the modeling very

293 complex and not easy to implement both from the point of view of the creation of the model (elements to be modeled,
294 hierarchy of structures, etc.), and from the point of view of the interpretation of the results obtained. From this
295 perspective, the graphic and analytical analysis of structures represents a valid and reliable tool for analyzing buildings
296 and investigating their structural behavior, serving as a starting point for further and more sophisticated analyses, both
297 local and global. Finally, it should be remembered that the above graphical methods of analysis are applied in the
298 context of a series of hypotheses about the materials, the constitutive bonds, the structural behaviors and the rupture
299 mechanisms that are clearly in favor of safety. Based on these considerations, this method makes it even more
300 interesting because, in the face of simplifications on the model, it's possible to obtain results that are both strongly
301 correlated with the behavior of the structures and able to guarantee good precision and adequate safety margins. By
302 enriching traditional methods of building knowledge with simplified and rapid graphic analysis methods, a further
303 useful element of knowledge is constituted, providing valuable information for planning interventions aimed at both
304 reuse and refunctionalization, as well as management and scheduled maintenance.

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6. Authors Contributions

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308 G.S. and A.T.; validation, E.L, G.S. and A.T.; formal analysis, E.L, V.P., G.S. and A.T.; investigation, E.L, M.M.,
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8. References

314 [1] Guida A, Porcari VD (2018) Prevention, monitoring and conservation for a smart management of the cultural
315   heritage. International Journal of Heritage Architecture, Vol. 2, No. 1 71–80 https://doi.org/10.2495/HA-V2-N1-
316   71-80

317 [2] Arfa FH, Zijlstra H, Lubelli B, Quist W (2022) Adaptive Reuse of Heritage Buildings: From a Literature Review
318   to a Model of Practice. The Historic Environment: Policy & Practice, 13(2), 148–170
319   https://doi.org/10.1080/17567505.2022.2058551

320 [3] Giuffrè A (1991) Letture sulla meccanica delle murature storiche. - Ed. Kappa, Roma

321 [4] Giuffrè A (1986) La meccanica dell'architettura: la statica. - Carocci editore, Roma

322 [5] Yang X, Grussenmeyer P, Koehl M, Macher H, Murtiyoso A, Landes T (2020) Review of built heritage modelling:
323   Integration of HBIM and other information techniques. Journal of Cultural Heritage, 46: 350-360.
324   https://doi.org/10.1016/j.culher.2020.05.008

325 [6] Tejedor B, Lucchi E, Bienvenido-Huertas D, Nardi I (2022) Non-destructive techniques (NDT) for the diagnosis of
326   heritage buildings: Traditional procedures and futures perspectives, Energy and Buildings, Volume 263, 2022,
327   112029, ISSN 0378-7788, https://doi.org/10.1016/j.enbuild.2022.112029

328 [7] Wang J, Yung YEN, (2023) Post-earthquake housing recovery with traditional construction: A preliminary review,
329   Progress, Disaster Science, Volume 18, 2023, 100283, ISSN 2590-0617,
330   https://doi.org/10.1016/j.pdisas.2023.100283

331 [8] Como M (2010) Statica delle costruzioni storiche in muratura. Archi, volte, cupole, architetture monumentali, edifici
332   sotto carichi verticali e sotto sisma. – Aracne, Roma

333 [9] Benvenuto E (1981) La scienza delle costruzioni e il suo sviluppo storico. - Ed. Sansoni, Firenze

334 [10] Heyman J (1995) The Stone Skeleton: Structural Engineering of Masonry Architecture. - Cambridge University
335   Press, Cambridge

336 **[11]** Heyman J (1969) The safety of masonry arches. In International Journal of Mechanical Sciences, 11, 4:363–385.
337   https://doi.org/10.1016/0020-7403(69)90070-8

338 **[12]** Heyman J (1982) The Masonry Arch. - Ellis Horwod, Chichester

339 **[13]** Heyman J (1996) Arches, vaults ad buttresses. Masonry Structures and Their Engineering. - Variorum, Aldershot

340 **[14]** Paradiso M, Tempesta G, Galassi S, Pugi F (2007) Sistemi voltati in muratura, teoria ed applicazioni. - DEI Editore,
341   Milano

342 **[15]** Leporelli E (2022) Architettura delle logge mercantili: linee guida per l'analisi, la valorizzazione e il recupero
343   sostenibile; Franco Angeli Editore, Milano, Italy.

344 **[16]** The "Straus 7" software is distributed in Italy by the company HSH srl of Padua.

345 **[17]** Deverdun G, and Université de Paris (1959) "Marrakech, Des Origines à 1912." Dissertation, Rabat: Editions
346   techniques nord-africaines

347 **[18]** Wilbaux Q (2001) La médina de Marrakech Formation des espaces urbains d'une ancienne capitale de Maroc.
348   L'Harmattan, Paris

349 **[19]** Lazzarini L, Borrelli E, Bouabdelli M, Antonelli F (2007) Insight into the conservation problems of the stone
350   building "Bab Agnaou", a XII cent. monumental gate in Marrakech (Morocco). Journal of Cultural Heritage,
351   Volume 8, Issue 3,2007. Pages 315-322, https://doi.org/10.1016/j.culher.2007.02.002