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MAXIMUM VARMA-TSALLIS ENTROPY BASED ON PORTFOLIO PROBLEM

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Abstract *This paper examines the application of entropy in portfolio optimization, focusing on the Shannon and Tsallis entropy frameworks. We use these nonparametric measures to construct optimal portfolios based on an empirical function for stock returns. The models are tested and compared using a dataset of eight stocks. Our empirical results show that the maximum entropy approach produces a well-diversified portfolio. Furthermore, analysis of the Varma- Tsallis model shows that a parameter value of 1 produces a portfolio similar to the optimal Tsallis entropy. Finally, we prove that as the Tsallis entropy parameter converges to 1, the optimal portfolio weights approach those derived from Shannon entropy.*

Keywords: *The Principle of Diversification, Portfolio Optimization, Maximum Entropy Principle, Renyi Entropy, Tsallis Entropy.*
AMS Classification: 94A17, 62B10.

1. Introduction

The objective of much financial research is optimal investing. Markowitz's (1952) mean-variance theory is one of the key theories in economics. According to Markowitz's theory, the market follows a normal distribution and investments should have the least amount of risk possible. In this methodology, sample moments are used to derive optimal portfolios. The Markowitz model uses the variance of data to measure financial risk. The principle of diversity is one of the fundamental ideas of this theory. This idea suggests that capital allocation should be spread among several stocks rather than placing all assets into a single stock.
The minimum average absolute value of deviations from the average model is one of the alternative techniques for determining the optimal portfolio. Because of the more diverse optimal portfolios created by this method, the investment risk in this model is lower than that in Markowitz model. Refer to Konno (2003), Konno and Kobayashi (1997), Erdas (2020), Ghahtarani and Najafi (2018), Kasenbacher et al. (2017) for more reading and details.
Some recent financial studies mentioned entropy can assess things that other statistical techniques cannot. According to Maasoumi and Racine (2002), entropy contains more information about the distribution than the variance does. Entropy was first described as an important indicator of portfolio optimization by Philippatos and Wilson in 1972. Two ways to measure uncertainty are entropy and vari-

ance. Shannon proposed the connection between the two scales. Entropy is only dependent on the probabilities of random variables and errors resulting from incorrect fitting of variable distributions do not affect entropy. Entropy is a more accurate indicator of dispersion than variance, according to many researchers. Reference Philippatos and Wilson (1972), Philippatos and Gressis (1975), Saxena (1983), Soofi (1997) and Popkov (2005) for more information. Many studies combine Markowitz theory and entropy to produce optimal portfolios. We can mention Philippatos and Gressis (1975), Hoskisson et al. (2006), Jana et al. (2009) and Palo (2016) among them. Optimal portfolios in this situation are more diverse and more in line with market realities for a simple reason: market investments are unpredictable and do not follow a normal distribution. Therefore, it is impossible to determine an investor's risk using the variance of observations. The family of functions known as Boltzmann-Gibbs entropy includes formulas for measuring discrete variables, uncertainty, and randomness in systems. Refer to Abdiel et al. (2016) for more detail. Tsallis (1988) developed a generalization of the Boltzmann-Gibbs entropy. The Tsallis entropy is less sensitive to the shape of the probability distribution than the Shannon entropy.

The Varma-Tsallis entropy is a generalization of the Tsallis entropy. Its properties allow it to be used for determining the probability density function of a random variable. Refer Varma (1966) and Romica et al. (2018) for mor information. Varma-Tsallis entropy has already been applied in portfolio optimization problems, confirming its practical relevance for robust financial modeling and risk-sensitive allocation strategies. Further details are provided in Batra and Taneja (2020), Batra and Taneja (2024) and Zhao et al. (2024). Varma-Tsallis entropy is a two-parameter measure that offers greater flexibility compared with Shannon entropy and Tsallis entropy (which is single-parametric). Varma-Tsallis entropy is, in fact, an extension of the Tsallis framework, providing enhanced capability for risk modeling. Moreover, as a more general measure, it can reproduce Tsallis and Renyi entropies as special cases, thereby ensuring theoretical consistency with well-established entropy metrics. Our primary objective in this paper was to clarify and demonstrate the theoretical relationships among Shannon, Tsallis, and Varma-Tsallis entropy models, as well as to evaluate them using the empirical function defined in the study. It is worth noting that more advanced frameworks and models have been examined in studies such as Batra and Taneja (2020), Batra and Taneja (2024), Zhao et al. (2024), Ahmed and Satchell (2024), Ardakani (2024) and Giunta et. al. (2024). Moving forward, we intend to expand our empirical analysis by incorporating these advanced models and indicators, thereby

further enhancing the practical applicability of the Varma-Tsallis entropy model. The paper's remaining sections are structured as follows: section 2 of this article discusses the economic models based on Hamza and Janssen's model and Markowitz's theory. In section 3, while stating the maximum Shannon, Renyi and Tsallis entropy principle, a new definition of entropy called Varma-Tsallis entropy is presented. The relationships between these entropies are examined. In the remaining sections of this part, we define the partitions of returns, and the optimal portfolio is determined using this definition for the maximum Shannon, Tsallis and Varma-Tsallis entropy. The optimal portfolio for the percentage returns of eight stocks over 54 days is determined in Section 4. We use economic models, the concept of maximum Shannon and Tsallis entropy for different parameter values. The findings are presented in a table. Section 5 has the conclusion.

2. An overview of common economic models used in portfolio selection

The challenge in constructing a portfolio lies in determining the optimal allocation of funds across each stock to achieve the highest expected return while minimizing investment risk, or to achieve a fixed level of risk with the lowest expected return. Markowitz (1952) was the first to address this issue and he argued that diversifying investments can reduce investment risk.

We define the portfolio as $X = (x_1, \ldots, x_n)$, where the components are the proportions invested in each stock and $X \in R^N$. $R_p = \sum_{i=1}^{n} x_i r_{it}$ is the portfolio return where r_{it} is the return rate of stock i time t. The time index t can represent a single day, a week, a month, a year, and so on. For T observation, the risk and expected return are as follows: $\Sigma = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \mu)(r_t - \mu)'$ and $\mu = \frac{1}{T}\sum_{t=1}^{T} r_t$ where $\mu \in R$ and $\Sigma \in R^{N\times N}$. Markowitz defined the relationship between expected return and risk. This is a convex quadratic problem with a linear constraint. If short selling is not permitted, the Mean-Variance Markowitz (MVP) model portfolio is expressed as follows:

$$\min_{x_i}(X'\mu - \frac{1}{2}X'\Sigma X),$$

within constraint

$$\sum_{i=1}^{n} x_i = 1, x_i \geq 0. \tag{1}$$

Relation (1) indicates that the investment condition has been completed.

Another investment strategy is to allocate all capital to stocks with the highest returns. This type of investment appears to be profitable but does not eliminate the possibility of bankruptcy.

Another type of investment that adheres to the diversification principle is the portfolio with minimum variance (GMVP). The goal of this investment is to select a portfolio that yields the highest expected return with the least amount of risk. In other words,

$$\min_{x_i} X'\Sigma X,$$

within constraint (1) and

$$X'\mu \geq \mu_0. \tag{2}$$

Hamza and Janssen (1996) demonstrated that completing the investment entirely is the lowest risk condition, according to condition

$$X'\mu + \frac{1}{T-1}\sum_{t=1}^{T}[\sum_{i=1}^{N}(r_{it}-\bar{r}_i)x_i]^2 \geq \mu_0. \tag{3}$$

A portfolio with the lowest absolute magnitude of error (MAD) is another type of investment model, based on the diversification principle. The following is the objective function. This problem's constraints are (1) and (2). Reference Murialdo et al. (2021) for more information,

$$\min_{x_i} \frac{1}{T-1}\sum_{t=1}^{T} \mid \sum_{i=1}^{N}(r_{it}-\bar{r}_i)x_i \mid .$$

3. Choosing the optimal portfolio based on the maximum entropy principle

In this section, we aim to determine the optimal portfolio weights (x_i) using the maximum entropy principle, while ensuring the satisfaction of at least one of the constraints stated in equations (2) and (3) in addition to constraint (1). In the following section, we define the empirical probability function for the returns to obtain the optimal weights.

Definition 3.1 *The Shannon entropy, commonly known as the entropy of the random variable X, is as follows if X is a discrete random variable with the probability function* $P(X) = (p_1(x), \ldots, p_n(x))$*, It's described as:*

$$H(X) = -\sum_{i=1}^{N} p(x_i)\ \log\ p(x_i).$$

The Shannon entropy is influenced solely by the probabilities of random variable X. The objective functions can be expressed as follows by combining the portfolio

with the maximum Shannon entropy (ME) principle:

$$\max_{x_i} -\sum_{i=1}^{n} x_i \log x_i, \tag{4}$$

Refer Yu et. al. (2014).
Assuming that there are N stocks in which we have invested, with m representing the length of time and r_{it} representing the return rate of stock i at time t. Let us denote the minimum expected return and maximum investment risk by μ_0 and σ_0^2. Constraints (2) and (3) can be rewritten as follows:

$$\sum_{i=1}^{N} x_i \bar{r}_i \geq \mu_0, \tag{5}$$

$$X'\Sigma X \leq \sigma_0^2. \tag{6}$$

According to the maximum Shannon entropy principle, which assumes that all feasible investments are equally likely, the optimal portfolio would have a uniform distribution. There are undoubtedly disadvantages to this kind of investment.

Definition 3.2 *Let X be a discrete random variable with probability function $P(X) = (p_1(x), \ldots, p_n(x))$. Renyi entropy, (Renyi, 1961), is equal to*

$$R_\alpha(X) = \frac{1}{1-\alpha} \log[\sum_{i=1}^{n} p(x_i)^\alpha)], \alpha > 0, \alpha \neq 1. \tag{7}$$

It is a generalized entropy of order α.

Definition 3.3 *The Tsallis entropy (Tsallis, 1988) is a generalization of Shannon entropy, which is described as*

$$S_\alpha(X) = \frac{1}{\alpha-1}[1 - \sum_{i=1}^{n} p(x_i)^\alpha], \alpha > 0, \alpha \neq 1, \tag{8}$$

where $P(X) = (p_1(x), \ldots, p_n(x))$ and α are respectively the discrete random variable probability function and the generalized parameret.

Shannon and Tsallis entropies are related in the way listed below

$$lim_{\alpha \to 1} S_\alpha(X) = H(X). \tag{9}$$

This formula was presented in Devi (2019)'s paper. The optimal portfolio produced by the maximum Shannon and Tsallis entropy is equivalent to $\alpha \to 1$ in the

next section using a real-world example.

The Markowitz problem can be combined with the maximum of Tsallis entropy under restrictions (1), (5), and (6) to create the objective function shown below:

$$S_\alpha(X) = \frac{1}{\alpha - 1}[1 - \sum_{i=1}^{n} x_i^\alpha]. \tag{10}$$

Presently, we are presenting a modern definition of entropy and demonstrating that it is a generalization of Shannon and Tsallis entropy.

Remark 3.4 *As we know, Tsallis entropy and Renyi entropy in equations (8) and (7) are connected via*

$$R_\alpha(X) = \frac{1}{1-\alpha} ln[1 + (1-\alpha)S_\alpha(X)],$$

where the result achieved trivially for Tsallis entropy also holds for Renyi entropy.

Definition 3.5 *Let X be discrete random variable with probability function* $P(X) = (p_1(x), \ldots, p_n(x))$. *Varma-Tsallis entropy is defined as:*

$$\begin{aligned} VT_{mv}(X) &= \frac{1}{m-v} \sum_{i=1}^{n} p(x_i)(1 - p(x_i)^{m+v-2}), \\ &= \frac{1}{m-v}[1 - \sum_{i=1}^{n} p(x_i)^{m+v-1}], m, v \neq 1. \end{aligned}$$

If

$$\alpha = m + v - 1, \tag{11}$$

we have

$$VT_{mv}(X) = \frac{1}{m-v} \sum_{i=1}^{n} p(x_i)(1 - p(x_i)^{\alpha-1}), \quad m, v \neq 1. \tag{12}$$

It follows from (11)

$$m = \alpha - v + 1.$$

Subsequently, the relation (12) can be written as follows:

$$VT_{mv}(X) = \frac{1}{\alpha - 2v + 1} \sum_{i=1}^{n} p(x_i)(1 - p(x_i)^{\alpha-1}), \quad m, v \neq 1. \tag{13}$$

The Varma-Tsallis entropy can be expressed as a coefficient of the Tsallis entropy, as shown in equation

$$VT_{mv}(X) = \frac{\alpha-1}{\alpha-2v+1}\frac{1}{\alpha-1}\sum_{i=1}^{n} p(x_i)(1-p(x_i)^{\alpha-1})$$
$$= \frac{\alpha-1}{\alpha-2v+1}S_\alpha, \quad m,v \neq 1.$$

In relation (13), if $v = 1$ is replaced, Tsallis entropy is the result. Refer to Tsallis (1988) and Varma (1966) for mor information.

Under (1), when at least one of (5) and (6) is used, the objective function resulting from the combination of Markowitz's problem and definition 4 is given by

$$S_\alpha(X) = \frac{1}{m-v}[1-\sum_{i=1}^{n} x_i(1-x_i^{m+v-2})]. \tag{14}$$

Definition 3.6 *Assuming that investments have been made in all n assets, we can define the return rate of stock i at time t as r_{it}. The random return matrix of n stocks is denoted by $R = (R_1,\ldots,R_n)$ and the matrix of R findings is denoted by $r = (r_1,\ldots,r_n)$. The expected return is calculated as $R_p = \sum_{i=1}^{n} x_i r_{it}$. For the observation matrix, we can define m different, equal-length partitions such that each return corresponds to only one partition. Let us denote one such partition as $A_1 = [a_0,a_1],\ldots,A_j = [a_{j-1},a_j], j = 2,\ldots,m$. The empirical probability for each event $R_P \in A_j$ is given by the formula*

$$\widehat{f}_T(r_1,\ldots,r_n;j) = \frac{1}{T}[1-\sum_{j=1}^{T} I(a_{j-1} < \sum_{i=1}^{n} x_i r_{it} \leq a_j). \tag{15}$$

Relation (15) can be inserted into the objective functions of portfolio selection optimization problems, the objective functions can be written as

$$\max_{x_i} -\sum_{i=1}^{N} \widehat{f}(r_1,\ldots,r_n;j) ln \widehat{f}(r_1,\ldots,r_n;j), \tag{16}$$

$$\max_{x_i} \frac{1}{\alpha-1}[1-\sum_{i=1}^{n} \widehat{f}^{\alpha}(r_1,\ldots,r_n;j)], \tag{17}$$

and

$$\max_{x_i} \frac{1}{m-v}[\sum_{i=1}^{n} \widehat{f}(r_1,\ldots,r_n;j)(1-\widehat{f}^{m+v-2}(r_1,\ldots,r_n;j)]. \tag{18}$$

and the optimization problem may be solved in accordance to constraints (5) and (6). In this case, the optimal results of the portfolios are superior to those of the previous because they are based on partitions. Choosing the number of these partitions has an impact on entropy even though it is optional. For the objective functions (16), (17) and (18), it can be noted that the entropy value will be zero if the length of the partitions is large enough to include all the observations, or if the distribution of portfolios is uniform, as is the case for the objective functions (4) and (10) when there is only one event in each partition.
As previously mentioned, we use entropy and average returns to quantify risk and expected return, respectively. Investors seek to maximize expected return while minimizing entropy, in order to achieve the highest return with the least amount of risk. This investment problem can be expressed as a multi-objective function. In other words,

$$\begin{cases} \max\limits_{x_i} \ -\sum_{i=1}^{n} x_i \log x_i \\ \\ \max\limits_{x_i} X'\mu. \end{cases} \qquad S.t. \quad \sum_{i=1}^{n} x_i = 1, x_i \geq 0 \tag{19}$$

The definition of the empirical probability function and the return's partition can be used to formulate equation (19). Consider the following problem:

$$\begin{cases} \max\limits_{x_i} \ -\sum_{i=1}^{n} \widehat{f} \log \widehat{f} \\ \\ \max\limits_{x_i} X'\mu. \end{cases} \qquad S.t. \quad \sum_{i=1}^{n} x_i = 1, x_i \geq 0 \tag{20}$$

Similarly, Tsallis and Varma-Tsallis entropy can be expressed using two-objective optimization functions (19) and (2), respectively. This can be done by simply substituting (10), (17), (14) and (18) for the first objective function in (19) and (2). In other words, this process can be summarized as

$$\begin{cases} \max\limits_{x_i} \ \frac{1}{\alpha-1}[1-\sum_{i=1}^{n} x_i^{\alpha}] \\ \\ \max\limits_{x_i} X'\mu, \end{cases} \qquad S.t. \quad \sum_{i=1}^{n} x_i = 1, x_i \geq 0$$

$$\begin{cases} \max\limits_{x_i} \ \frac{1}{\alpha-1}[1-\sum_{i=1}^{n} x_i^{\alpha}] \\ \\ \max\limits_{x_i} X'\mu, \end{cases} \qquad S.t. \quad \sum_{i=1}^{n} x_i = 1, x_i \geq 0$$

$$\begin{cases} \max\limits_{x_i} \frac{1}{m-v}[1-\sum_{i=1}^{n} x_i(1-x_i^{m+v-2})] \\ \\ \max\limits_{x_i} X'\mu, \end{cases} \qquad S.t. \quad \sum_{i=1}^{n} x_i = 1, x_i \geq 0$$

$$\begin{cases} \max\limits_{x_i} \frac{1}{\alpha-1}[1-\sum_{i=1}^{n} \widehat{f}^{\alpha}] \\ \\ \max\limits_{x_i} X'\mu, \end{cases} \qquad S.t. \quad \sum_{i=1}^{n} x_i = 1, x_i \geq 0$$

$$\begin{cases} \max\limits_{x_i} \frac{1}{m-v}[\sum_{i=1}^{n} \widehat{f}(1-\widehat{f}^{m+v-2})] \\ \\ \max\limits_{x_i} X'\mu. \end{cases} \qquad S.t. \quad \sum_{i=1}^{n} x_i = 1, x_i \geq 0$$

There are several solutions for multi-objective optimization functions. While a solution may be the best in terms of one objective function, it may also be the worst in terms of another. Thus, multi-objective problems usually have a set of solutions that are difficult to compare. However, by combining multiple objective functions into one, the problem can have a singular solution, which offers a significant benefit. Thus, the following formulation can be used to represent the two-objective optimization functions discussed above. The parameter β is a measure of risk tolerance, and it has little impact on the results.

$$\min_{x_i}[-\sum_{i=1}^{n} x_i \log x_i - \beta X'\mu],$$

$$\min_{x_i}[\frac{1}{\alpha-1}[1-\sum_{i=1}^{n} x_i^{\alpha}] - \beta X'\mu],$$

$$\min_{x_i}[\frac{1}{m-v}[\sum_{i=1}^{n} x_i[1-x_i^{m+v-2}] - \beta X'\mu],$$

$$\min_{x_i}[-\sum_{i=1}^{n} \widehat{f}(r_1,\ldots,r_n;j)\log \widehat{f}(r_1,\ldots,r_n;j) - \beta X'\mu],$$

$$\min_{x_i}[\frac{1}{\alpha-1}[1-\sum_{i=1}^{n} \widehat{f}(r_1,\ldots,r_n;j)^{\alpha}] - \beta X'\mu],$$

$$\min_{x_i}[\frac{1}{m-v}[\sum_{i=1}^{n} \widehat{f}(r_1,\ldots,r_n;j)[1-\widehat{f}(r_1,\ldots,r_n;j)^{m+v-2}] - \beta X'\mu].$$

4. Numerical example

The data of this article is related to the daily prices of 8 stocks from 2020.1.1 to 2023.12.31 and was collected from YahooFinance. The choice of eight stocks over 54 days was not due to methodological constraints but rather a practical consideration to ensure clarity and readability of the tables and typesetting within the manuscript. Importantly, the proposed entropy-based framework is general and can be applied to larger datasets and across different markets. Daily stock returns were calculated as follows:

$$r_{it} = \frac{p_{i,t}}{p_{i,t-1}} - 1,$$

Where r_{it} and $p_{i,t}$ are the returns and the i-th stock price in the t-th period, respectively.

Table 1 displays the sample's mean and variance.

Tables 2 through 4, respectively, show the results of the optimal portfolio for Markowitz, maximum Shannon, and Tsallis entropy models with and without exclusion. ($\mu_0 = -0.00015, \sigma_0^2 = 0.00087$)

Table 1: The sample's mean and variance

Return Parametrs	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Mean	0.00030	0.00055	0.00050	0.00051	-0.0024	-0.00001	0.00009	-0.0004
Variance	0.000033	0.0012	0.0004	0.00017	0.0018	0.00039	0.00028	0.0014

The findings of Table 2 demonstrate the exact equality of Hamza and Janssen (1996) and the Markowitz models under constraints (1) and (2).

Table 2: The optimal portfolio based on Markowz, Hamza and Janssen.

Model / *Constraints*	$\min_{x_i} 0.5X'\Sigma X$							
$X'\mu \geq \mu_0, X'\Sigma X \leq \sigma_0^2$	0.125	0.1248	0.125	0.1252	0.1249	0.125	0.1251	0.1248
$X'\mu + \frac{1}{T-1}\sum_{t=1}^{T}[\sum_{i=1}^{N}(r_{it}-\bar{r}_i)x_i]^2 \geq \mu_0$	0.125	0.1248	0.125	0.1252	0.1249	0.125	0.1251	0.1248

Table 3: The optimal portfolio based on the maximum entropy principle

Constraint / *Max*	$X'\mu \geq \mu_0, \sum_{i=1}^{n} x_i = 1, x_i \geq 0, v = 1, \alpha = 0.99, 1.01$							
$-\sum_{i=1}^{n} x_i \log x_i$	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
$\frac{1}{\alpha-1}[1-\sum_{t=1}^{n} x_i^{\alpha}]$	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
$\frac{1}{m-v}\sum_{t=1}^{n} x_i[1-x_i^{m+v-2}]$	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
$-\sum_{i=1}^{n} \widehat{f} \log \widehat{f}$	0.1380	0.1459	0.1444	0.1447	0.0520	0.1279	0.1312	0.1156
$\frac{1}{\alpha-1}[1-\sum_{t=1}^{n} \widehat{f^{\alpha}}]$	0.1380	0.1459	0.1444	0.1447	0.0520	0.1279	0.1312	0.1156
$\frac{1}{m-v}\sum_{t=1}^{n} \widehat{f}[1-\widehat{f}^{m+v-2}]$	0.1380	0.1459	0.1444	0.1447	0.0520	0.1279	0.1312	0.1156

Based on Table 3, it can be inferred that while the optimal portfolio distribution generated by the maximum Shannon, Tsallis, and Varma-Tsallis entropy models is uniform, the distribution of portfolio values produced by the empirical function of returns is more diverse. Additionally, as α approaches 1, ($\alpha \to 1$), the optimal values predicted by the maximum Tsallis entropy model and the maximum Varma-Tsallis entropy model when $v = 1$ agree with the maximum Shannon entropy values. The results obtained for values greater than μ_0 and less than σ_0^2 are identical to the omitted values in the table, which have been excluded for the sake of brevity.

Tabel 4 states that in order to have an optimal portfolio based on the Markowitz model, all investments should be made in the second and fifth stocks. Alternatively, a uniform distribution of capital among all eight stocks can also yield an optimal portfolio according to the Markowitz model, combined with Shannon, Tsallis, and Varma-Tsallis entropy. Because the portfolios in this instance do not adhere to the principle of investment diversification, the results of these three models are far from reflecting the reality of the stock market. On the other hand, market distribution is not frequently uniform.

The results obtained from the combination of the constraints of the Markowitz model with each of the three entropies under the empirical probability function show that in order to have an optimal portfolio, investments should be made in only 6 stocks out of the eight available stocks, so that the highest and lowest weight of the investments are The order should be in shares 2 and 6.

Furthermore, it can be noted that the optimal portfolio values obtained from maximum Shannon and Tsallis entropies are equivalent to $\alpha \to 1$ and $v = 1$, when subject to common constraints.

5. Conclusion

In this paper, we computed the optimal portfolios based on conventional eco-

Table 4: The optimal portfolio based on the maximum entropy principle and the empirical probability function

Constraints / *min*	$\sum_{i=1}^{n} x_i = 1, x_i \geq 0$							
$[\frac{1}{2}X'\Sigma X - X'\mu]$	0	0.040	0	0	0.0598	0	0	0
$[-\sum_{i=1}^{n} x_i \log x_i - X'\mu]$	0.1250	0.1250	0.1250	0.1250	0.1249	0.1250	0.1250	0.1249
$[\frac{1}{\alpha-1}[1-\sum_{t=1}^{n} x_i^{\alpha}] - X'\mu]$	0.1250	0.1250	0.1250	0.1250	0.1249	0.1250	0.1250	0.1249
$[\frac{1}{m-v}[\sum_{t=1}^{n} x_i[1-x_i^{m+v-2}] - X'\mu]$	0.1250	0.1250	0.1250	0.1250	0.1249	0.1250	0.1250	0.1249
$[-\sum_{i=1}^{n} \widehat{f} \log \widehat{f} - X'\mu]$,	0.1566	0.2596	0.2404	0.2441	0	0.0307	0.0689	0
$[\frac{1}{\alpha-1}[1-\sum_{t=1}^{n} \widehat{f^{\alpha}}] - X'\mu]$,	0.1566	0.2596	0.2404	0.2441	0	0.0307	0.0689	0
$[\frac{1}{m-v}[\sum_{t=1}^{n} \widehat{f}[1-\widehat{f^{m+v-2}}] - X'\mu]$	0.1566	0.2596	0.2404	0.2441	0	0.0307	0.0689	0

nomic models (mean-variance and mean absolute deviation) as well as several entropy-based models. The classical Markowitz mean-variance model defines risk solely in terms of variance, which in practice may lead to concentrated portfolios with allocations heavily skewed toward a limited number of assets_a limitation that has also been noted in previous studies. In contrast, our empirical comparisons demonstrate that entropy-based models, particularly the Varma-Tsallis framework, consistently generate portfolios with higher levels of diversification relative to the mean-variance benchmark. This improvement is reflected in the distribution of weights across assets. The results demonstrated that the optimal portfolio determined by the minimum variance model, under the minimum return, and the Hamza and Janssen model are equal if the investment is complete. Since the stock return is a random variable, it frequently deviates from the normal distribution. The maximal entropy concept is non-parametric. The combined Markowitz and entropy model's optimal portfolios outperform the Markowitz model in terms of performance. These models' results are more realistic and better reflect what happens in the actual market. Because most of these portfolios adhere to the maximization of the entropy principle or the Markowitz and entropy combination models, they are less risky than the ideal portfolios proposed by Markowitz. The three maximum entropy principles of Shannon, Tsallis and Varma-Tsallis provide exactly the same optimal results for $v = 1$, neighborhood values of $\alpha = 1$. Consequently, the optimal investment weights can be determined based on Tsallis entropy since it outperforms Shannon entropy. Entropy is applied in finance to maximize diversification and risk management through non-parametric portfolio optimization. By employing Shannon and Tsallis entropy, we have optimized the portfolio using empirical probability of stock returns. This analysis confirms that as $\alpha = 1$ approaches 1, $\alpha \to 1$, the Tsallis and Varma-Tsallis portfolios (with $v = 1$) converge to the Shannon entropy solution, establishing theoretical conti-

nuity. Thus, entropy offers a robust, diversity-focused framework, distinct from purely empirical allocation strategies.

Availability of Data

The data used in this paper is publicly available online. It can be accessed at YahooFinance.

Declarations

- Ethics approval Not applicable.
- Consent to participate Not applicable.
- Consent for publication Not applicable.
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