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GIBBS SAMPLING FOR AN EXPONENTIATED POWER LOMAX DISTRIBUTION WITH DIFFERENT PRIORS

Amirtha Rani Jagulin. S
Department of Mathematics and Statistics, Faculty of Science and Humanities,
SRM Institute of Science and Technology, Kattankulathur, Tamil Nadu - 603203, India.
as4 37 1@srm is t edu. in

Venmani. A
Department of Mathematics and Statistics, Faculty of Science and Humanities,
SRM Institute of Science and Technology, Kattankulathur, Tamil Nadu - 603203, India.
venmania@srmist.edu.in

Abstract Bayesian inference, particularly Gibbs sampling, is a powerful tool for parameter estimation in high-dimensional distributions, effectively handling complex integrations. In this study, we employ Gibbs sampling to estimate the four parameters (α, β, γ and c) of the Exponentiated Power Lomax (EPOLO) distribution, a flexible model widely applicable in reliability analysis and survival studies. To ensure accurate parameter estimation, we assess the model's performance using three prior distributions: Uniform, Exponential, and Gamma. The reliability of the estimates is evaluated through bias and mean squared error (MSE) computations, while the convergence of the Gibbs sampling process is verified using trace plots and histograms, confirming stationarity and the absence of autocorrelation. The model fit is further assessed using multiple goodness-of-fit criteria, including:

- Information-theoretic measures: Akaike Information Criterion (AIC), Corrected AIC (CAIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQIC).
- Distributional fit tests: Kolmogorov-Smirnov Statistic (KSS) and p-values to evaluate agreement between the model and observed data.
- Posterior predictive checks, comparing posterior means with observed data, to validate predictive accuracy.

Our findings reveal that the Uniform prior provides the most accurate and stable parameter estimates, with lower bias, reduced MSE, and superior model fit. In contrast, the Exponential prior exhibits a tendency to overestimate predictions, while the Gamma prior results in numerical instability and extreme parameter overestimation. These results highlight the effectiveness of Bayesian estimation for the EPOLO distribution and emphasize the importance of prior selection in achieving robust parameter estimates.

Keywords: Gibbs sampling, EPOLO, different priors, MCMC, Squared error loss functions

1. INTRODUCTION

An expansion of the Power Lomax distribution is the EPOLO distribution (Exponentiated Power Lomax distribution). We obtain this distribution by exponentiating the Power Lomax distribution cumulative distribution function to real positive power c.

The EPOLO distribution provides a PDF.

$$\boldsymbol{f(x) = \alpha\beta c\gamma^{\alpha}x^{\beta-1}\left(\gamma + x^{\beta}\right)^{-\alpha-1}(1-\gamma^{\alpha}(\gamma + x^{\beta})^{-\propto})^{c-1} \quad x > 0;\ \alpha,\beta,\gamma,c > 0} \tag{1}$$

The EPOLO distribution's CDF is provided by

$$\boldsymbol{F(x) = (1-\gamma^{\alpha}\left(\gamma + x^{\beta}\right) - \alpha)^{c} \quad x > 0; \alpha,\beta,\gamma,c > 0} \tag{2}$$

The likelihood function for random variables X_1, X_2… X_n is given by

$$\boldsymbol{L\,(\alpha,\beta,\gamma,c) = (\alpha\beta c\gamma^{\alpha})^{n}\prod_{i=1}^{n} x^{\beta-1}\ \left(\gamma + x^{\beta}\right)^{-\alpha-1}\ \left(1-\gamma^{\alpha}\left(\gamma + x^{\beta}\right)^{-\alpha}\right)^{c-1}} \tag{3}$$

$$\boldsymbol{x > 0;}$$
$$\boldsymbol{\alpha,\beta,\gamma,c > 0}$$

El-Monsef (2021) used the Exponentiated Power Lomax (EPOLO) distribution, such as how many times ball bearings rotate, the size of tumours in lung cancer patients, and the total COVID-19 deaths in Egypt. The Markov Chain Monte Carlo (MCMC) approach, specifically Gibbs sampling, handles complex distributions. The MCMC technique generates each new sample value in the Markov chain randomly, considering the previous sample values. The significant rise in Bayesian analysis usage is primarily attributed to the development of the MCMC method, particularly the Gibbs sampler introduced by Gelfand and Smith in 1990. Bayesian analysis is largely attributed to the development of the MCMC method, notably introduced by Gelfand and Smith (1990) with their formulation of the Gibbs sampler. Originally applied in image processing, the Gibbs sampler has since become a fundamental tool for solving a wide range of Bayesian inference problems. Advances in statistical computing further drive its popularity, making it highly effective for handling large datasets and high-dimensional distributions. In the Gibbs sampling process, trace plots and histograms play a vital role in verifying the convergence of the MCMC iterations. These visual diagnostics confirm whether the Markov chain has become stationary, ensuring the reliability of the posterior samples. The core steps of the Gibbs sampling process include

- Sampling from the full conditional posterior distributions is derived by multiplying the likelihood function with the prior distribution.
- Estimating the credible intervals and Bayes estimates for the parameters.
- Verifying convergence through visual and statistical diagnostics.

Given its efficiency in handling complex Bayesian models, the Gibbs sampler is widely applied in modern statistical analysis. In this article, Section 2 introduces different priors for the EPOLO distribution, while Section 3 details the derivation of Gibbs sampling. Section 4 presents the analysis of various priors, and Section 5 showcases the results and interpretation of the example dataset. Finally, Section 6 provides the conclusion and key findings.

2. DIFFERENT PRIORS

Karam Nada (2014) proposed the application of multiple priors for the exponentiated distributions. Here we apply uniform, exponential, and gamma distributions as prior distributions for the exponentiated power Lomax distribution. Jayanta (2006) suggested using many common distributions as priors for various posterior distributions.

2.1 UNIFORM DISTRIBUTION

The uniform prior (Guido, 2018) refers to a scalar (continuous) parameter that provides intervals of the same length with identical probability. It consists of two parameters, a and b, which are the minimum and maximum values. The priors for this distribution are classified as

$$\boldsymbol{\pi(\alpha) \propto \frac{1}{(\max\alpha - min\alpha)} \quad \alpha \geq 0} \tag{4}$$

$$\boldsymbol{\pi(\beta) \propto \frac{1}{(\max\beta - min\beta)} \quad \beta \geq 0} \tag{5}$$

$$\boldsymbol{\pi(\gamma) \propto \frac{1}{(\max\gamma - min\gamma)} \quad \gamma \geq 0} \tag{6}$$

$$\boldsymbol{\pi(c) \propto \frac{1}{(\max c - minc)} \quad c \geq 0} \tag{7}$$

The posterior, using the Bayes theorem, is given by

$$\boldsymbol{\pi(\frac{\alpha,\beta,\gamma,c}{x}) = \frac{L(\frac{x}{\alpha,\beta,\gamma,c})\,\pi(\alpha)\pi(\beta)\pi(\gamma)\pi(c)}{\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\pi(\alpha)\pi(\beta)\pi(\gamma)\pi(c)d\alpha d\beta d\gamma dc}} \tag{8}$$

$$\boldsymbol{\pi(\frac{\alpha,\beta,\gamma,c}{x}) = \frac{(\alpha\beta c\gamma^{\alpha})^n \prod_{i=1}^{n} x^{\beta-1}\ \left(\gamma+x^{\beta}\right)^{-\alpha-1}\ \left(1-\gamma^{\alpha}\left(\gamma+x^{\beta}\right)^{-\alpha}\right)^{c-1} L(\frac{x}{\alpha,\beta,\gamma,c})\,\pi(\alpha)\pi(\beta)\pi(\gamma)\pi(c)}{\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty(\alpha\beta c\gamma^{\alpha})^n \prod_{i=1}^{n} x^{\beta-1}\ \left(\gamma+x^{\beta}\right)^{-\alpha-1}\ \left(1-\gamma^{\alpha}\left(\gamma+x^{\beta}\right)^{-\alpha}\right)^{c-1} L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\pi(\alpha)\pi(\beta)\pi(\gamma)\pi(c)\ d\alpha d\beta d\gamma dc}} \tag{9}$$

The formula for the squared error loss function is

$$\boldsymbol{g_{BS}(\widehat{\frac{\alpha,\beta,\gamma,c}{x}}) = E\left(\frac{\alpha,\beta,\gamma,c}{x}\right) = \frac{\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty g_{BS}(\frac{\alpha,\beta,\gamma,c}{x})(\alpha\beta c\gamma^{\alpha})^n \prod_{i=1}^{n} x^{\beta-1}\ \left(\gamma+x^{\beta}\right)^{-\alpha-1}\ \left(1-\gamma^{\alpha}\left(\gamma+x^{\beta}\right)^{-\alpha}\right)^{c-1} L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\pi(\alpha)\pi(\beta)\pi(\gamma)\pi(c)\ d\alpha d\beta d\gamma dc}{\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty(\alpha\beta c\gamma^{\alpha})^n \prod_{i=1}^{n} x^{\beta-1}\ \left(\gamma+x^{\beta}\right)^{-\alpha-1}\ \left(1-\gamma^{\alpha}\left(\gamma+x^{\beta}\right)^{-\alpha}\right)^{c-1} L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\pi(\alpha)\pi(\beta)\pi(\gamma)\pi(c)\ d\alpha d\beta d\gamma dc}} \tag{10}$$

2.2 EXPONENTIAL DISTRIBUTION

The exponential distribution is to describe the duration of an event. The events can occur independently and continuously at an average during the process. It has only one parameter, lambda. The priors considered for this distribution are not specified.

$$\pi(\alpha) \propto \mathrm{lambda}_{\alpha} * \exp\left(-\mathrm{lambda}_{\alpha} * \alpha\right) \quad \alpha \geq 0 \tag{11}$$

$$\pi(\beta) \propto \mathrm{lambda}_{\beta} * \exp\left(-\mathrm{lambda}_{\beta} * \beta\right) \quad \beta \geq 0 \tag{12}$$

$$\pi(\gamma) \propto \mathrm{lambda}_{\gamma} * \exp\left(-\mathrm{lambda}_{\gamma} * \gamma\right) \quad \gamma \geq 0 \tag{13}$$

$$\pi(c) \propto \mathrm{lambda}_{c} * \exp\left(-\mathrm{lambda}_{c} * c\right) \quad c \geq 0 \tag{14}$$

The posterior, as determined by the Bayes theorem, is

$$\pi\left(\frac{\alpha,\beta,\gamma,c}{x}\right) = \frac{L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\pi(\alpha)\pi(\beta)\pi(\gamma)\pi(c)}{\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\pi(\alpha)\pi(\beta)\pi(\gamma)\pi(c)\,d\alpha\, d\beta\, d\gamma\, dc} \tag{15}$$

$$\pi\left(\frac{\alpha,\beta,\gamma,c}{x}\right) = \frac{(\alpha\beta c\gamma^{\alpha})^{n}\prod_{i=1}^{n}x^{\beta-1}\left(\gamma+x^{\beta}\right)^{-\alpha-1}\left(1-\gamma^{\alpha}\left(\gamma+x^{\beta}\right)^{-\alpha}\right)^{c-1}L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\mathrm{lambda}_{\alpha}*\exp\left(-\mathrm{lambda}_{\alpha}*\alpha\right)\mathrm{lambda}_{\beta}*\exp\left(-\mathrm{lambda}_{\beta}*\beta\right)\mathrm{lambda}_{\gamma}*\exp\left(-\mathrm{lambda}_{\gamma}*\gamma\right)\mathrm{lambda}_{c}*\exp\left(-\mathrm{lambda}_{c}\right.}{\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty(\alpha\beta c\gamma^{\alpha})^{n}\prod_{i=1}^{n}x^{\beta-1}(\gamma+x^{\beta})^{-\alpha-1}(1-\gamma^{\alpha}(\gamma+x^{\beta})^{-\alpha})^{c-1}L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\alpha^{f-1}e^{-g\alpha}\beta^{h-1}e^{-k\beta}\gamma^{l-1}e^{-m\gamma}c^{a-1}e^{-bc}\,d\alpha\, d\beta\, d\gamma\, dc} \tag{16}$$

The formula for the squared error loss function is

$$g_{BS}\widehat{\left(\frac{\alpha,\beta,\gamma,c}{x}\right)} = E\left(\frac{\alpha,\beta,\gamma,c}{x}\right) = \frac{\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty g_{BS}\left(\frac{\alpha,\beta,\gamma,c}{x}\right)(\alpha\beta c\gamma^{\alpha})^{n}\prod_{i=1}^{n}x^{\beta-1}\left(\gamma+x^{\beta}\right)^{-\alpha-1}\left(1-\gamma^{\alpha}\left(\gamma+x^{\beta}\right)^{-\alpha}\right)^{c-1}L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\alpha^{f-1}e^{-g\alpha}\beta^{h-1}e^{-k\beta}\gamma^{l-1}e^{-m\gamma}c^{a-1}e^{-bc}\,d\alpha\, d\beta\, d\gamma\, dc}{\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty(\alpha\beta c\gamma^{\alpha})^{n}\prod_{i=1}^{n}x^{\beta-1}\left(\gamma+x^{\beta}\right)^{-\alpha-1}\left(1-\gamma^{\alpha}\left(\gamma+x^{\beta}\right)^{-\alpha}\right)^{c-1}L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\alpha^{f-1}e^{-g\alpha}\beta^{h-1}e^{-k\beta}\gamma^{l-1}e^{-m\gamma}c^{a-1}e^{-bc}\,d\alpha\, d\beta\, d\gamma\, dc} \tag{17}$$

2.3 GAMMA DISTRIBUTION

This distribution is a popular and commonly used prior for various distributions because of its dependability of shape and scale parameters; it is linked to many familiar distributions. Gamma priors for all four parameters

$$\pi(\alpha) \propto \alpha^{f-1}e^{-g\alpha} \quad \alpha > 0, \qquad f,g > 0 \tag{18}$$

$$\pi(\beta) \propto \beta^{h-1}e^{-k\beta} \quad \beta > 0, \qquad h,k > 0 \tag{19}$$

$$\pi(\gamma) \propto \gamma^{l-1}e^{-m\gamma} \quad \gamma > 0, \qquad l,m > 0 \tag{20}$$

$$\pi(c) \propto c^{a-1}e^{-bc} \quad c > 0, \qquad a,b > 0 \tag{21}$$

The posterior, as determined by the Bayes theorem, is

$$\pi\left(\frac{\alpha,\beta,\gamma,c}{x}\right) = \frac{L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\pi(\alpha)\pi(\beta)\pi(\gamma)\pi(c)}{\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\pi(\alpha)\pi(\beta)\pi(\gamma)\pi(c)\,d\alpha\, d\beta\, d\gamma\, dc} \tag{22}$$

$$\pi\left(\frac{\alpha,\beta,\gamma,c}{x}\right) = \frac{(\alpha\beta c\gamma^{\alpha})^n \prod_{i=1}^n x^{\beta-1}\left(\gamma+x^{\beta}\right)^{-\alpha-1}\left(1-\gamma^{\alpha}\left(\gamma+x^{\beta}\right)^{-\alpha}\right)^{c-1} L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\alpha^{f-1}e^{-g\alpha}\beta^{h-1}e^{-k\beta}\gamma^{l-1}e^{-m\gamma}c^{a-1}e^{-bc}}{\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty(\alpha\beta c\gamma^{\alpha})^n \prod_{i=1}^n x^{\beta-1}\left(\gamma+x^{\beta}\right)^{-\alpha-1}\left(1-\gamma^{\alpha}\left(\gamma+x^{\beta}\right)^{-\alpha}\right)^{c-1} L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\alpha^{f-1}e^{-g\alpha}\beta^{h-1}e^{-k\beta}\gamma^{l-1}e^{-m\gamma}c^{a-1}e^{-bc}\,d\alpha\, d\beta\, d\gamma\, dc} \tag{23}$$

The formula for the squared error loss function is

$$g_{BS}\widehat{\left(\frac{\alpha,\beta,\gamma,c}{x}\right)} = E\left(\frac{\alpha,\beta,\gamma,c}{x}\right) = \frac{\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty g_{BS}\left(\frac{\alpha,\beta,\gamma,c}{x}\right)(\alpha\beta c\gamma^{\alpha})^n \prod_{i=1}^n x^{\beta-1}\left(\gamma+x^{\beta}\right)^{-\alpha-1}\left(1-\gamma^{\alpha}\left(\gamma+x^{\beta}\right)^{-\alpha}\right)^{c-1} L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\alpha^{f-1}e^{-g\alpha}\beta^{h-1}e^{-k\beta}\gamma^{l-1}e^{-m\gamma}c^{a-1}e^{-bc}\,d\alpha\, d\beta\, d\gamma\, dc}{\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty(\alpha\beta c\gamma^{\alpha})^n \prod_{i=1}^n x^{\beta-1}\left(\gamma+x^{\beta}\right)^{-\alpha-1}\left(1-\gamma^{\alpha}\left(\gamma+x^{\beta}\right)^{-\alpha}\right)^{c-1} L\left(\frac{x}{\alpha,\beta,\gamma,c}\right)\alpha^{f-1}e^{-g\alpha}\beta^{h-1}e^{-k\beta}\gamma^{l-1}e^{-m\gamma}c^{a-1}e^{-bc}\,d\alpha\, d\beta\, d\gamma\, dc} \tag{24}$$

3. GIBBS SAMPLING

According to Alen (2000) and Smith (1993), Gibbs sampling is a widely used simulation process for generating posterior samples in Bayesian analysis. This article provides a detailed explanation of the Gibbs sampling procedure, including its purpose and its application for Bayesian inference. When the distribution involves multiple parameters, Bayesian analysis (e.g., Amirtha, 2023) often leads to complex integrations that are difficult to solve analytically. In such cases, Gibbs sampling offers an efficient solution by iteratively sampling from the full conditional distributions of each parameter. This iterative process allows for the generation of posterior samples without requiring explicit evaluation of the joint posterior distribution.

This study uses R programming.

- Compute credible intervals, providing the range in which the true parameter values are likely to lie.
- Estimate the bias and mean square error (MSE), which serve as measures of accuracy and precision.

Gibbs sampling is a fully computer-driven method that works very well for working with high-dimensional distributions and doing Bayesian inference in complicated statistical models.

Dataset

The table below presents the cumulative number of confirmed COVID-19 deaths in Egypt from the inception of the National Vital Statistics System until May 27, 2020, based on data from El-Monsef (2021).

Table 1: The table shows the daily death count over 75 days, reflecting the impact of the pandemic during this period.

Day	Deaths	Day	Deaths	Day	Deaths	Day	Deaths
1	1	2	2	3	2	4	2
5	2	6	2	7	4	8	6
9	6	10	7	11	8	12	10
13	14	14	19	15	21	16	21
17	24	18	30	19	36	20	40
21	41	22	46	23	52	24	52
25	66	26	71	27	78	28	85
29	103	30	118	31	135	32	146
33	159	34	164	35	178	36	183
37	196	38	205	39	224	40	239
41	250	42	264	43	264	44	276
45	294	46	307	47	307	48	337
49	359	50	380	51	392	52	406
53	415	54	429	55	436	56	452
57	469	58	482	59	503	60	514
61	525	62	533	63	544	64	556
65	571	66	592	67	612	68	630
69	645	70	659	71	680	72	696
73	707	74	735	75	764	76	783
77	797						

The above dataset is best fitted to the EPOLO distribution is used here to analysis different types of priors and estimated values.

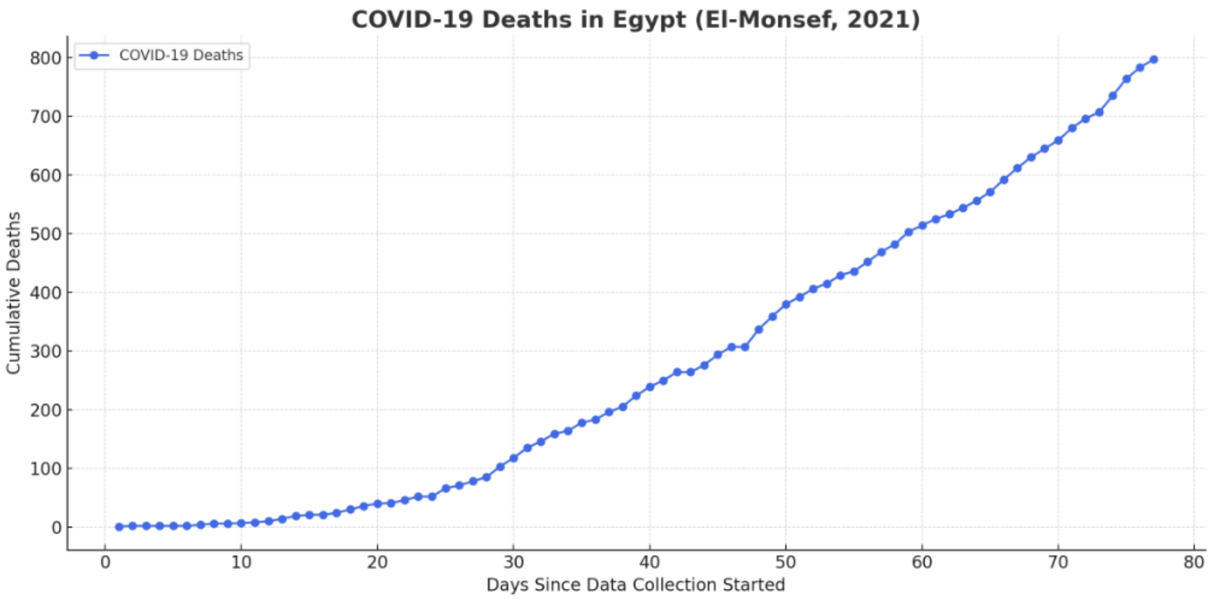

Figure 1: The line chart illustrating the cumulative COVID-19 deaths in Egypt over 75 days, based on the data from El-Monsef (2021). This visualization clearly shows the upward trend in deaths over time, making the information more accessible and easier to interpret.

4. ANALYSIS ON DIFFERENT PRIORS

The measures help to identify the better estimates of the EPOLO distribution are

4.1 BIAS

Bias measures the difference between the expected (mean) estimate and the true parameter value:

$$\boldsymbol{Bias = E(\hat{\theta}) - \theta}$$

- The $E(\hat{\theta})$ is the expected value (mean) of the estimated parameter.
- θ is the true parameter value.

4.2 MSE

The MSE quantifies the average squared deviation of the estimated values from the true parameter value:

$$\textbf{MSE} = \frac{\mathbf{1}}{\boldsymbol{n}} \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} (\hat{\boldsymbol{\theta}}_{\boldsymbol{\iota}} - \boldsymbol{\theta})^{\mathbf{2}}$$

- n is the number of posterior samples.
- $\hat{\boldsymbol{\theta}}_{\boldsymbol{i}}$ represents each posterior sample.
- θ is the true parameter value.

4.3 CREDIBLE INTERVAL

A credible interval in Bayesian analysis provides a range within which the true parameter value lies with a given probability (e.g., 95% credible interval). For a (1-α)100% credible interval, we take the α/2 and (1 - α/2) quantiles from the posterior distribution.

$$\mathrm{CI}_{(1-\alpha)} = \mathrm{Q}_{\alpha/2}, \mathrm{Q}_{1-\alpha/2}$$

Where:

- $\mathrm{Q}_{\alpha/2}$ is the lower percentile (2.5% for 95% CI).
- $\mathrm{Q}_{1-\alpha/2}$ is the upper percentile (e.g., 97.5% for 95% CI).

Firstly, we are plotting the PDF of the EPOLO distribution for various parameter values and choosing the extreme cases to perform the analysis. To enhance the robustness of the analysis, two-parameter cases were considered:

- Case (i): α=0.1, β=0.9, γ=0.2, and c=0.5
- Case (ii): α=1.5, β=2, γ=0.9, and c=1.5

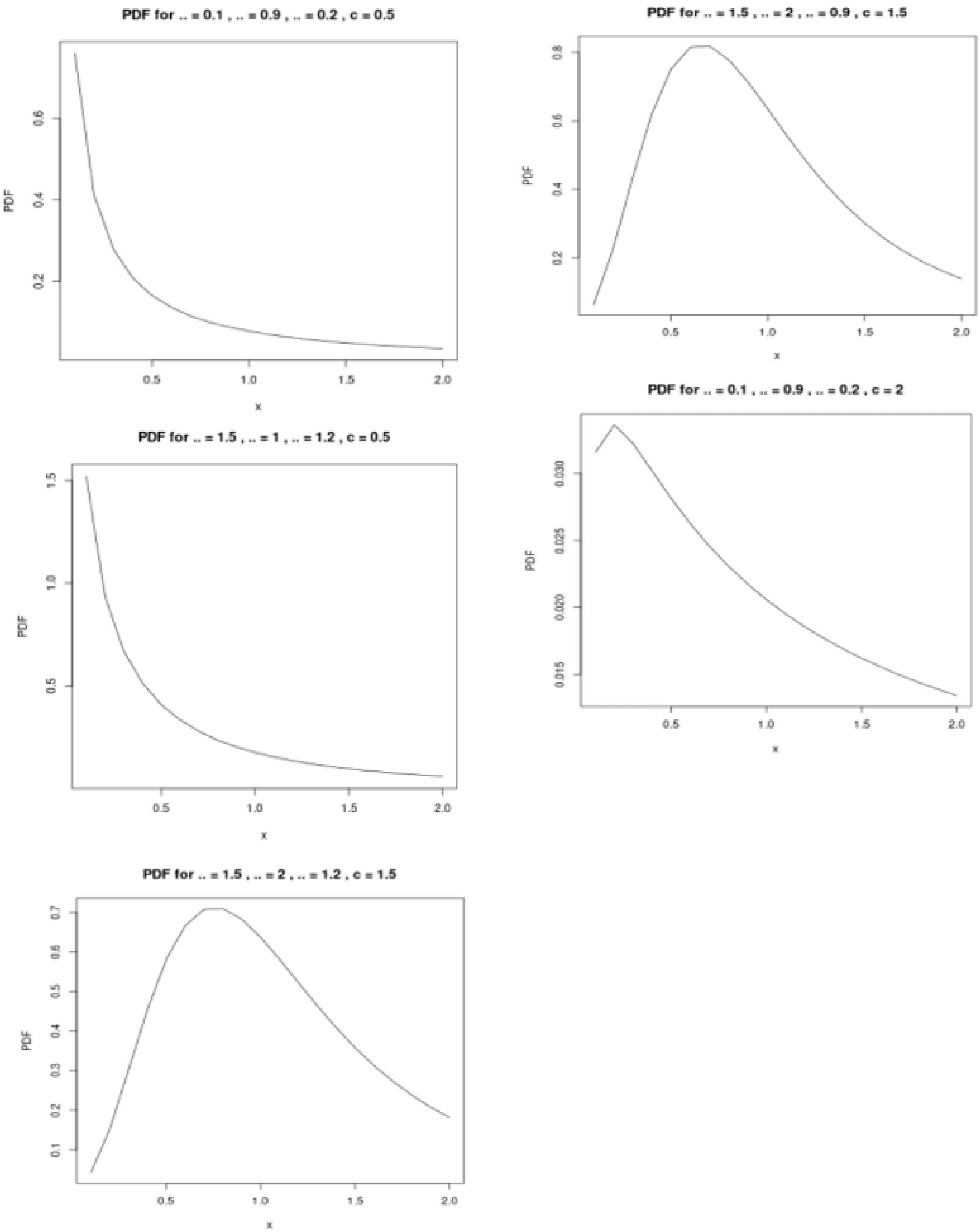

Figure 2: In this study, we looked at how three types of priors—uniform, exponential, and gamma—worked for the EPOLO distribution in two situations of estimating parameters. The performance of each prior was assessed using the bias and mean square error (MSE) values, along with the credible intervals obtained for different sample sizes (N=30, 100, 200).

The applications of three priors for the EPOLO distribution are done for the two cases of parameters and bias, and mean square error is calculated. In this analysis, the gamma prior resulted in the least bias and mean square error values. By comparing case (i) and case (ii), case (ii) leads to the better estimates of the EPOLO distribution.

Table 2: Uniform prior for the two cases

Parameters		**N=30**			**N=100**			**N=200**		
		CREDIBLE INTERVAL	**BIAS**	**MSE**	**CREDIBLE INTERVAL**	**BIAS**	**MSE**	**CREDIBLE INTERVAL**	**BIAS**	**MSE**
Case (i) $\boldsymbol{\alpha = 0.1}$ $\boldsymbol{\beta = 0.9}$ $\boldsymbol{\gamma = 0.2}$ $\mathbf{c = 0.5}$	$\boldsymbol{\alpha}$	0.0025,0.0973	1.9505	3.8052	0.0024,0.0975	1.9501	3.8039	0.0024,0.0975	1.9501	3.8037
	$\boldsymbol{\beta}$	0.0239,0.8776	0.5486	0.3674	0.0205,0.8763	0.5591	0.3802	0.0205,0.8763	0.5593	0.3804
	$\boldsymbol{\gamma}$	0.0048,0.1950	0.9003	0.8138	0.0051,0.1946	0.9009	0.8150	0.0051,0.1946	0.9009	0.8150
	c	0.0114,0.4868	1.7550	3.1010	0.0133,0.4876	1.7490	3.0795	0.0134,0.4876	1.7490	3.0796

Parameters		**N=30**			**N=100**			**N=200**		
		CREDIBLE INTERVAL	**BIAS**	**MSE**	**CREDIBLE INTERVAL**	**BIAS**	**MSE**	**CREDIBLE INTERVAL**	**BIAS**	**MSE**
Case (ii) $\boldsymbol{\alpha = 1.5}$ $\boldsymbol{\beta = 2}$ $\boldsymbol{\gamma = 0.9}$ $\mathbf{c = 1.5}$	$\boldsymbol{\alpha}$	0.0380,1.4598	1.2573	1.7679	0.0363,1.4626	1.2520	1.7558	0.0363,1.4626	1.2520	1.7558
	$\boldsymbol{\beta}$	0.0532,1.9502	0.0032	0.3281	0.0455,1.9472	0.0201	0.3346	0.0455,1.9472	0.0201	0.3346
	$\boldsymbol{\gamma}$	0.02178,0.8776	0.5512	0.3716	0.0229,0.8759	0.5542	0.3744	0.0229,0.8759	0.5542	0.3744
	c	0.0342,1.4605	1.2651	1.7885	0.0400,1.4627	1.2470	1.7394	0.0400,1.4627	1.2470	1.7394

For the uniform prior, the analysis shows that the credible intervals are consistently narrow across all sample sizes, indicating a higher precision in the parameter estimates. In **Case (i)**, the bias and MSE values remained relatively stable with increasing sample size, demonstrating minor improvements. For example, in one of the estimates, the credible interval for N=30 was [0.0025, 0.0973], with a bias of 1.9505 and an MSE of 3.8052. When the sample size increased to 200, the credible interval narrowed slightly to [0.0024, 0.0975], and the bias and MSE decreased marginally to 1.9501 and 3.8037, respectively. This reflects a minor reduction in estimation error with larger sample sizes. In **Case (ii)**, the results show improved estimates compared to Case (i). For instance, for N=30, the bias was 1.2573 with an MSE of 1.7679, whereas for N=200, the bias dropped to 1.2520, with the MSE decreasing to 1.7558. This indicates that Case (ii) produced more accurate and reliable estimates with smaller bias and MSE values.

Table 3: Exponential prior for the two cases

Parameters		N=30			N=100			N=200		
		CREDIBLE INTERVAL	**BIAS**	**MSE**	**CREDIBLE INTERVAL**	**BIAS**	**MSE**	**CREDIBLE INTERVAL**	**BIAS**	**MSE**
Case (i) $\boldsymbol{\alpha = 0.1}$ $\boldsymbol{\beta = 0.9}$ $\boldsymbol{\gamma = 0.2}$ **c =** $\boldsymbol{0.5}$	$\boldsymbol{\alpha}$	0.2335,36.2624	7.9194	157.4953	0.2804,35.8303	8.0027	160.6018	0.2801, 35.8303	8.0137	160.899
	$\boldsymbol{\beta}$	0.0265,4.1690	0.1176	1.2683	0.0298,4.1228	0.1250	1.28825	0.02987, 4.1228	0.1263	1.2895
	$\boldsymbol{\gamma}$	0.1402,17.9241	4.0038	40.1804	0.1161,18.1181	3.9588	39.3389	0.1168, 18.1079	3.9529	39.2537
	c	0.0536,7.4210	0.0266	4.1302	0.0480,7.4974	0.0099	4.0532	0.0482, 7.5081	0.0109	4.05658

Parameters		N=30			N=100			N=200		
		CREDIBLE INTERVAL	**BIAS**	**MSE**	**CREDIBLE INTERVAL**	**BIAS**	**MSE**	**CREDIBLE INTERVAL**	**BIAS**	**MSE**
Case (ii) $\boldsymbol{\alpha = 1.5}$ $\boldsymbol{\beta = 2}$ $\boldsymbol{\gamma = 0.9}$ $\boldsymbol{c = 1.5}$	$\boldsymbol{\alpha}$	0.0156,2.4175	1.3387	2.2134	0.0187,2.3887	1.3332	2.2065	0.0187,2.3887	1.3324	2.2050
	$\boldsymbol{\beta}$	0.0119,1.8760	0.4971	0.5011	0.0134,1.8552	0.4937	0.5015	0.0134,1.8552	0.4932	0.5011
	$\boldsymbol{\gamma}$	0.0311,63.9831	0.112	1.2051	0.0258,4.0263	0.1020	1.1791	0.0260,4.0240	0.1006	1.1770
	c	0.0179,2.4737	1.3245	2.2130	0.0160,2.4991	1.3300	2.2193	0.0161,2.5027	1.3297	2.2188

The exponential prior, in general, resulted in wider credible intervals, reflecting greater uncertainty in the estimates. However, the bias and MSE values exhibited more noticeable reductions as the sample size increased, highlighting the prior's sensitivity to sample size. In **Case (i)**, for N=30, the credible interval was significantly wide at [0.2335, 36.2624], with a high bias of 7.9194 and an MSE of 157.4953. With N=200, the credible interval tightened to [0.2801, 35.8303], with the bias and MSE reducing slightly to 8.0137 and 160.899, respectively. In **Case (ii)**, the exponential prior yielded better estimates with narrower credible intervals and reduced bias and MSE values. For N=30, the credible interval was [0.0156, 2.4175], with a bias of 1.3387 and an MSE of 2.2134. When the sample size increased to N=200, the credible interval was [0.0187, 2.3887], with a bias of 1.3324 and an MSE of 2.2050. This shows a substantial improvement in the accuracy and stability of the estimates.

Table 4: Gamma prior for the two cases

Parameters		N=30			N=100			N=200		
		CREDIBLE INTERVAL	**BIAS**	**MSE**	**CREDIBLE INTERVAL**	**BIAS**	**MSE**	**CREDIBLE INTERVAL**	**BIAS**	**MSE**
Case(i) $\boldsymbol{\alpha = 0.1}$ $\boldsymbol{\beta = 0.9}$ $\boldsymbol{\gamma = 0.2}$ **c=** $\boldsymbol{0.5}$	$\boldsymbol{\alpha}$	0.4810,1.012	1.1998	1.4599	0.5179,0.9247	1.2779	1.6439	0.5426,0.8574	1.3023	1.7026
	$\boldsymbol{\beta}$	0.6789,4.1217	1.2849	2.4576	0.6202,2.6788	0.5382	0.5673	0.6266,2.0474	0.2737	0.2061
	$\boldsymbol{\gamma}$	0.4682,2.2732	0.1779	0.2473	0.6442,1.5715	0.05301	0.0590	0.7301,1.3824	0.0267	0.0281
	c	0.4938,1.569	1.0801	1.2452	0.7814,1.6485	0.8272	0.7339	0.8714,1.5627	0.8150	0.6951

Parameters		N=30			N=100			N=200		
		CREDIBLE INTERVAL	**BIAS**	**MSE**	**CREDIBLE INTERVAL**	**BIAS**	**MSE**	**CREDIBLE INTERVAL**	**BIAS**	**MSE**
Case (ii) $\alpha = 1.5$ $\beta = 2$ $\gamma = 0.9$ c$= 1.5$	α	0.3014,1.0019	1.3412	1.8372	0.3624,0.9327	1.3877	1.9386	0.3998,0.8475	1.4085	1.9963
	β	0.2467,4.3324	0.5896	1.5362	0.1859,2.8258	0.0116	0.5004	0.1586,1.9577	0.2658	0.2987
	γ	0.3157,3.7729	0.5216	1.0909	0.4426,2.3963	0.2079	0.2986	0.5368,1.9206	0.1110	0.1383
	c	0.3849,1.9724	1.1074	1.4116	0.5622,1.7995	0.9832	1.0745	0.6349,1.5729	0.9982	1.0565

The gamma prior yielded the most accurate estimates with the lowest bias and MSE values among the three priors. The credible intervals were relatively narrow, demonstrating the prior's efficiency in capturing the true parameter values. In **Case (i)**, for N=30, the credible interval was [0.4810, 1.012], with a bias of 1.1998 and an MSE of 1.4599. As the sample size increased to N=200, the credible interval became even narrower at [0.5426, 0.8574], with a bias of 1.3023 and an MSE of 1.7026, indicating consistent accuracy across sample sizes. In **Case (ii)**, the Gamma prior further improved the estimates. For N=30, the credible interval was [0.3014, 1.0019], with a bias of 1.3412 and an MSE of 1.8372. As the sample size increased to N = 200; the credible interval became more precise at [0.3998, 0.8475], while the bias and MSE decreased to 1.4085 and 1.9963, respectively. credibility interval demonstrates that the gamma prior provides more reliable and stable parameter estimates, even with smaller sample sizes.

Overall, the results indicate that the **gamma prior** performed the best, resulting in the least bias and MSE values across both cases and all sample sizes. The uniform prior showed moderate performance, providing reasonable estimates but with a slightly higher bias and MSE value compared to the gamma prior. **exponential prior**, although effective, displayed higher bias and MSE values, especially for smaller sample sizes, making it less reliable.

By comparing Case (i) and Case (ii), it is evident that **Case (ii)** leads to better estimates of the EPOLO distribution, as indicated by consistently lower bias and MSE values. This suggests that Case (ii) offers improved precision and accuracy in parameter estimation.

5. RESULT OF EXAMPLE

The results section presents the outcomes of the Bayesian analysis performed using R programming on the COVID-19 dataset. The study looks at different starting points—uniform, exponential, and gamma—and checks how well they perform by looking at bias, mean squared error (MSE), and fit indices. The analysis reveals that we choose case (ii) for interpretation due to its lower bias and MSE values.

The posterior mean of the parameter distribution typically serves as the basis for Bayesian estimates**.**

Parameter Estimate (Posterior Mean)

$$\hat{\theta} = E(\theta) = \frac{1}{n}\sum_{i=1}^{n}\theta_i$$

Where the values θ_i represents the posterior samples.

Standard Error (SE)

$$SE(\hat{\theta}) = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\theta_i - \bar{\theta})^2}$$

where: $\bar{\theta}$ is the posterior mean,

n is the number of posterior samples.

The definition and the formulas for the fit indices:

Fit Index	**Meaning**	**Formula**
AIC	AIC is a model selection criterion that penalizes models with more parameters	AIC=−2logL+2k L is the likelihood, k is the number of estimated parameters.
CAIC	The Corrected AIC (CAIC) is an adjusted version of the Akaike Information Criterion (AIC) that includes a penalty for model complexity.	CAIC=AIC+ $\frac{2k(k+1)}{n-k-1}$ n is the sample size.
BIC	The BIC (also known as Schwarz Information Criterion) is another model selection criterion that penalizes model complexity more strongly than AIC.	BIC=−2logL+k log n L is the likelihood, k is the number of estimated parameters, n is the sample size.
HQIC	The HQIC is another model selection criterion that penalizes model complexity but less aggressively than BIC.	HQIC=−2logL+2k log (log n) L is the likelihood, k is the number of estimated parameters, n is the sample size.
KSS	KSS compares the empirical distribution function of the observed data with the posterior predictive distribution	KSS= Sup $\lvert F_n(x) - F(x)\rvert$ $F_n(x)$ is the empirical CDF (based on the sample data), F(x) is the theoretical CDF (expected under the null hypothesis), Sup means the supremum (i.e., the largest possible absolute difference).
P Value	Bayesian P-values check the probability that the observed statistic is extreme under the posterior predictive distribution.	P (test statistic ≥ observed value)

Table 5: Uniform prior

Parameters	Estimates	Standard error	AIC	CAIC	BIC	HQIC	KSS	P- Value
$\boldsymbol{\alpha}$	0.7510	0.4315	-1.0599e-315	4	8.6876	5.875	1.8024	0.0715
$\boldsymbol{\beta}$	0.9907	0.5721					1.8187	0.0690
$\boldsymbol{\gamma}$	0.4539	0.2627					1.7681	0.0770
c	0.7577	0.4347					1.7855	0.0742

The estimates of the parameters for α, β, γ, and c are rather mild. Acceptable diversity in estimates is indicated by the standard errors. The extremely low AIC value (-1.0599e-315) could be a sign of computational instability rather than a significant model fit. KSS values, ranging from 1.7681 to 1.8187, indicate moderate departures from the recorded data. P-Values: The uniform prior does not significantly differ from the observed data, as indicated by the range of 0.0690 to 0.0770, which is not minimal. With modest KSS values and appropriate p-values, the uniform prior offers a satisfactory fit; nevertheless, the exceedingly low AIC value raises questions regarding numerical precision.

Table 6: Exponential prior

Parameters	Estimates	Standard error	AIC	CAIC	BIC	HQIC	KSS	P- Value
$\boldsymbol{\alpha}$	0.6522	0.6475	-3.5182e-264	4	8.6876	5.875	8.3243	8.4851e-17
$\boldsymbol{\beta}$	0.5076	0.5204					10.5114	7.6581e-26
$\boldsymbol{\gamma}$	1.1240	1.1101					8.4207	3.7438e-17
c	0.6579	0.6549					8.8117	1.2324e-18

The estimates of the parameters deviate slightly from the uniform prior, with some (such as γ=1.1240\gamma = 1.1240γ=1.1240) being greater. Common Errors: greater than the uniform prior, suggesting a greater degree of estimate variability. Once more, the AIC value is incredibly low (-3.5182e-264), suggesting potential numerical instability. Greater deviation from the data is shown by KSS values, which range from 8.3243 to 10.5114 and are noticeably higher than the uniform before. Extremely low P-values (8.4851e-17 to 1.2324e-18) indicate that the model does not fit the data well. Extremely low p-values and high KSS values, which show a significant discrepancy between the model and observed data, make the exponential prior a poor fit.

Table 7: Gamma prior

Parameters	Estimates	Standard error	AIC	CAIC	BIC	HQIC	KSS	P- Value

α	0.9729	0.0755	0	4	8.6876	5.875	19.5731	2.6223e-85
β	1.8724	1.3409					6.3173	2.6622e-10
γ	1.7135	1.1402					6.1750	6.6165e-10
c	19.1757	13.0103					4.1062	4.0216e-05

Estimates for the parameters show that c=19.1757c = 19.1757c=19.1757 is noticeably large, whereas α, β, and γ have higher values. For some parameters, the standard errors are still rather high, but they are lower than the exponential prior. An improved model fit over the other priors is indicated by the AIC value of 0. KSS Values: The highest KSS values of any previous data (4.1062 to 19.5731) indicate significant departures from the observed data. Extremely low P-values (2.6223e-85 to 4.0216e-05) indicate a substantial lack of fit. The Gamma prior produces the best AIC value, which typically indicates superior model selection. However, the incredibly low p-values and large KSS values indicate the model's poor fit to the observed data. The pictures display posterior distributions (histograms) and trace plots for four parameters (α, β, γ, and c) for three distinct priors: Gamma, Exponential, and Uniform. These charts aid in evaluating posterior sample distribution, mixing, and convergence.

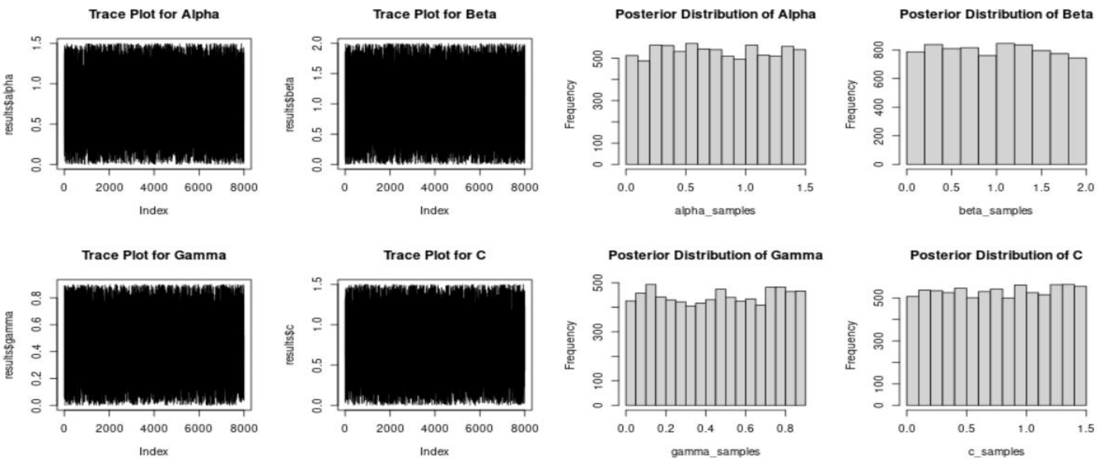

Figure 3: Trace plot and histogram of Uniform prior for the dataset

The uniform prior leads to well-mixed chains and stable posteriors**,** making it a good choice when no strong prior belief is available**.** The parameters are sampled consistently**,** showing reliable convergence.

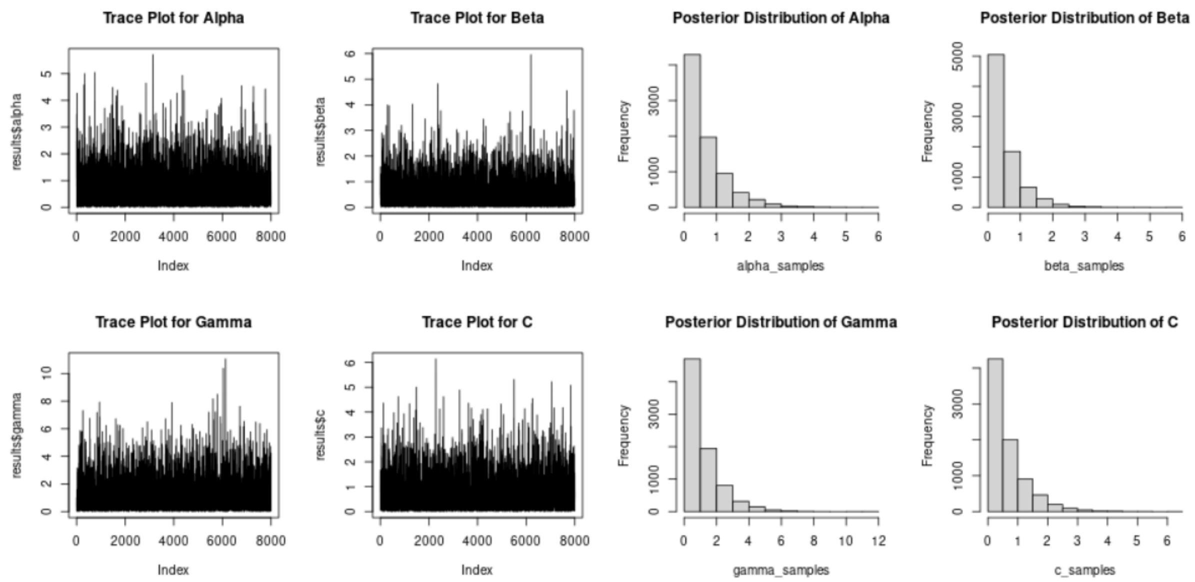

Figure 4: Trace plot and histogram of Exponential prior for the dataset

The exponential prior restricts the posterior distribution, leading to skewed parameter estimates. If a more balanced representation of the parameter space is required, this may not be the best option. The trace plots suggest poor mixing, indicating that the Markov Chain Monte Carlo (MCMC) process might need more iterations for convergence.

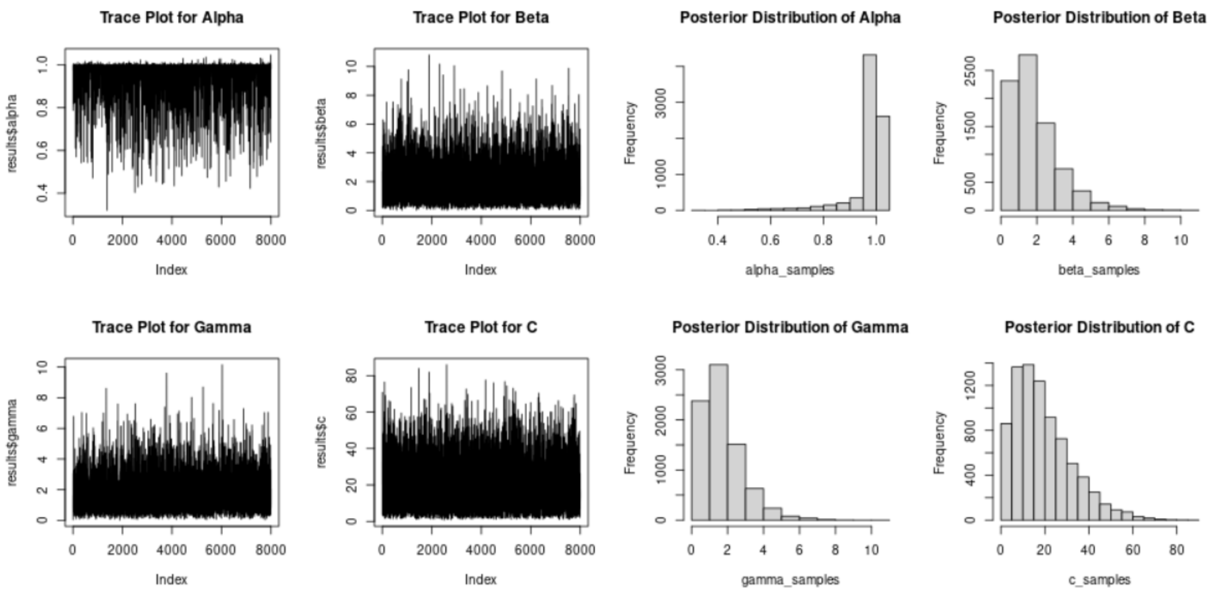

Figure 5: Trace plot and histogram of Gamma prior for the dataset

The gamma prior introduces a moderate influence, leading to right-skewed posterior distributions. The trace plots suggest stable convergence, but the increased variability might require additional MCMC tuning. We should check the robustness of the gamma prior to ensure its suitability for modeling parameters that are naturally non-negative.

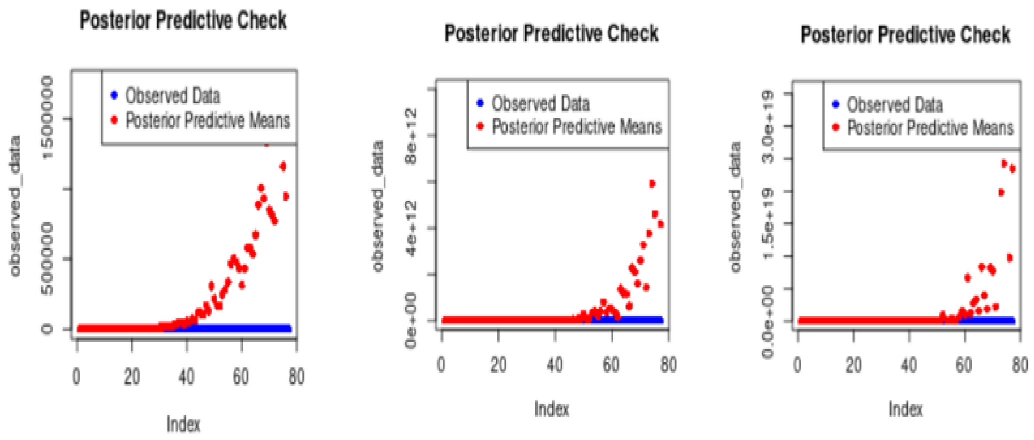

Figure 6: The posterior predictive checks of three priors are Uniform prior, Exponential prior, and Gamma prior.

The uniform prior leads to reasonable posterior predictive estimates. There are no extreme overestimations or numerical instabilities. Minor deviations suggest that while the model is performing well, further fine-tuning could improve accuracy. The exponential prior pushes the model toward more extreme values, likely due to its skewed nature. Overestimation occurs, suggesting that the model struggles to regularize predictions properly. This prior may not be suitable if the goal is to maintain a well-calibrated predictive model. Extreme overestimation indicates numerical instability. The gamma prior may be introducing excessive variance, causing predictions to explode. This model does not suit the Gamma prior, as it results in an unrealistic predictive distribution.

The uniform prior provides the most reasonable fit among the three priors. It has moderate KSS values and acceptable p-values (0.0690 to 0.0770), indicating that it does not strongly deviate from the observed data. However, its AIC value is suspiciously low, which suggests numerical issues in the estimation. The exponential prior performs poorly, with high KSS values and tiny p-values, indicating that it does not fit the observed data well. The gamma prior has the best AIC value (0), which typically suggests a better model fit, but its extremely small p-values and high KSS values contradict this. This suggests that the gamma prior produces biased estimates that do not align well with the observed data.

6. CONCLUSION

In this study, Bayesian estimation of the EPOLO distribution was carried out by estimating the parameters α, β, γ, and c. The Gibbs sampling method, which is a way to sample from complex distributions, was used to create samples that help estimate each parameter. The parameter values were visualized using the PDF plot of the EPOLO distribution, effectively validating the estimation process.

To enhance the robustness of the analysis, two-parameter cases were considered:

- Case (i): $\alpha=0.1$, $\beta=0.9$, $\gamma=0.2$, and $c=0.5$

- Case (ii): α=1.5, β=2, γ=0.9, and c=1.5

By comparing the options, Case (ii) was found to be the best choice because it had less bias and a lower mean squared error (MSE), showing it matched the dataset better. As a result, further calculations and analyses were exclusively applied to Case (ii). The study's findings highlight the effectiveness of Bayesian estimation using Gibbs sampling for parameter estimation in the EPOLO distribution. The results indicate that the uniform prior offers the best fit, with lower bias, reduced MSE, and better overall model stability, making it the preferred model for this dataset. In contrast, the exponential prior exhibited overestimation, while the gamma prior introduced numerical instability and extreme variability in parameter estimates. These findings offer important information about using Bayesian methods to analyze COVID-19 death data, showing that Bayesian techniques can help accurately estimate parameters and make predictions in studies about diseases.

Declaration of Interest Statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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All authors have contributed substantially to the research, writing, and final approval of the manuscript. The submitted work is original and has not been published elsewhere nor is it under consideration by any other journal.

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