Il Colle di Galileo

Conformal Field Theories and Renormalization Group Flows in Dimensions *d*>2

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Abstract. This workshop dealt with the recent advances in the solution of strongly coupled theories above two dimensions that are conformal invariant. In particular, the conformal bootstrap approach was shown to predict scaling dimensions and operator-product coefficients in relevant theories such as the three-dimensional Ising model and four-dimensional supersymmetric gauge theories. Other non-perturbative results on renormalization group flows were also discussed. The workshop enjoyed a wide participation, including experts, PhD students and interested researchers.

Keywords. Conformal invariance, conformal bootstrap, renormalization group flows, non-perturbative results, Ising model in three dimensions, integrability in supersymmetric theories.

Scientific background

Conformal invariant field theories (CFTs) describe the massless fixed points in the space of quantum field theories and are connected among themselves by renormalization group (RG) flows. Understanding the fixed points and the pattern of flows at a deeper, non-perturbative level is one of the basic goals of theoretical physics, and could be relevant to the understanding of numerous physical problems. In two spacetime dimensions, conformal theories can be exactly solved, owing to the existence of an infinite-dimensional group of symmetries. Starting from the eighties, these two-dimensional exact results have been successfully applied to many physical systems, ranging from statistical mechanics models to condensed matter setups and to string theory.

This GGI workshop was devoted to conformal theories in dimensions greater than two, whose physical applications are of no less importance. Three-dimensional theories and flows among them describe the critical, near-critical and crossover behavior of condensed matter systems and statistical models. Four-dimension-

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al CFTs with strong dynamics are often relevant for various scenarios of physics beyond the Standard Model, meant to solve puzzles related to flavor physics, the hierarchy problem, etc. Of course, Quantum Chromodynamics is the strongly coupled *4d* flow *par excellence*. Six-dimensional supersymmetric CFTs play an important role in string theory, describing the physics of *M5* branes.

Until about ten years ago, conformal symmetry in more than two dimensions was believed to be of limited use, and few results were known, often dating back to the sixties and early seventies. A breakthrough took place when the **conformal bootstrap** approach was reconsidered and used to obtain a number of striking results in three and four dimensional theories. Over the last six years, a new, very active domain of research has been developing. The present strong interest in the subject was testified by the major participation in the workshop, which can be considered as one of the most successful activities of the Galileo Galilei Institute, in terms of number of participants, scientific contributions *et cetera*.

The conformal bootstrap approach deals with the exact correlation functions of the theory and analyzes the constraints following from crossing symmetry: namely, it imposes the associativity of the operator algebra of conformal fields (Operator Product Expansion - OPE). Ideally, this approach is similar to the axiomatic S-matrix theory of the sixties, but is far more predictive as well as more technically involved. It is believed that the conformal bootstrap equations uniquely define the CFT up to the choice of symmetries and few other conditions. Of course, the bootstrap approach was very successful for two-dimensional CFTs, leading to exact solutions for correlators, once combined with the infinite conformal symmetries. In higher dimensions, the constraints of crossing symmetry and unitarity have been used to derive bounds on the operator spectrum and on the OPE structure constants. Since these bounds were often obtained by numerical analysis of the bootstrap equations, one of the main themes of the workshop was that of developing analytic tools.

More generally, the workshop aimed at discussing the non-perturbative methods that investigate the dynamics of CFTs and renormalization group flows, and their use for making concrete predictions. Outlined below are a number of closely-related research topics that were developed during the workshop.

Ising model in *3d*

The numerical implementation of the bootstrap has already been very successful for understanding CFTs. Their existence can be predicted using standard techniques (perturbative renormalization group flows, Monte Carlo simulations on the lattice, and experiments), but their strongly coupled nature makes it difficult to make precise predictions. The prime example is the recent determination of the critical exponents of the *3d* Ising model with an accuracy exceeding that obtained earlier by the epsilon-expansion and Monte Carlo simulations. In the same bootstrap computation, the central charge and several structure constants of its operator algebra were also determined for the first time.

Classification of CFTs

The conformal bootstrap may in principle lead to a classification of CFTs in *d>2*. While this goal is currently beyond reach for non-supersymmetric CFTs, plausible classification conjectures can be formulated for supersymmetric CFTs, especially for theories with extended supersymmetry. It has been shown that bootstrap equations for any *N=2* supersymmetric CFT in *d=4* contain an exactly solvable subsector described by a *2d* chiral algebra. The same is true for *(2,0)* theories in *d=6*. These results show great promise and may be the cornerstone of a future classification of such theories. The study of supersymmetric CFTs also raises several new structural questions. The landscape of such theories generically exhibits two interesting phenomena – the possibility of spontaneous conformal symmetry breaking and the presence of conformal manifolds associated with exactly marginal deformations - which are not observed without supersymmetry.

Conformal boundary conditions

The conformal bootstrap can also be used to study CFTs in the presence of conformal defects. Considering the theory in a half-space with conformally invariant boundary conditions is a particular case, but one can consider other defects, such as interfaces, or defects of larger codimension, such as Wilson lines in conformal gauge theories or the line defect in the *3d* Ising model, the characteristics of which have been recently understood. For higher-dimensional CFTs, the discussion centered on the possibility of classifying conformally invariant boundary conditions in analogy with the *d=2* case of Cardy's states.

AdS/CFT correspondence

Under certain conditions, a *d*-dimensional CFTs may be thought of as defining a theory of quantum gravity in *(d+1)*-dimensional AdS space, via the AdS/CFT correspondence. On the one hand, this relation let us to address general structural questions of quantum gravity via CFT techniques, which are on more solid ground. On the other hand, powerful quantum gravity intuition based on black holes, high energy graviton scattering, etc., can be used to predict some general properties of CFTs. Recently, this connection has been used to conjecture (and later prove) results about the structure of the operator spectrum at high spin values, that are valid for any CFT in *d>2*.

Integrability in *4d*

Recent studies of *4d* non-Abelian gauge theories via the AdS/CFT correspondence and other methods have shown instances of integrability, i.e. exact solvability. For example, they allowed the calculation of anomalous dimensions of gauge-invariant composite operators. Furthermore, the $N = 4$ supersymmetric theories are conformally invariant and believed to be integrable in the planar limit, and some $N = 2$ theories possess integrable protected sectors. Understanding four-dimensional integrability, in comparison with two-dimensional integrability, is a major problem of theoretical and mathematical physics, which can be tackled directly by the conformal bootstrap.

Irreversibility of RG flows

The impressive progress in CFTs in $d>2$ has been accompanied by a renewed understanding of the RG flows connecting them. Some very general results have recently been obtained regarding the irreversibility of the flow (the so-called "a-theorem" in *4d* and "F-theorem" in *3d*). It is still an open question whether there are analogous results in six dimensions, and some exploratory results were presented at the workshop. Another interesting direction is the use of flows for deforming and studying the UV fixed point itself.

Strongly coupled RG flows

A further fundamental question is what it means, operationally, to have a flow originating at a strongly coupled fixed point. It is known that the very beginning of the flow can be well captured by conformal perturbation theory. Another, numerical technique, the Truncated Conformal Space Approach introduced by Al. Zamolodchikov, can describe the flow all the way down to the IR fixed point. Until very recently, this has been successfully applied only for $d=2$. Traditionally, strongly coupled flows in $d>2$ have been studied via lattice Monte Carlo techniques; other techniques, such as the gap equation or the functional renormalization group, have had limited success. That the Truncated Conformal Space Approach could provide interesting results in $d>2$ was widely discussed during the workshop.

The Workshop

This workshop was the last in a series of meetings on conformal theories in $d>2$, called "Back to the Bootstrap", that were first organized in 2011 and 2012 at the Perimeter Institute, in 2013 at CERN, in 2014 at the KITP Santa Barbara, in 2014 in Porto and in May 2015 at the Weizmann Institute. These were specialized two-week programs with a participation of a maximum of 50. The workshop at the

GGI was the first big event, in terms both of the duration and attendance, and it coincided in a timely manner with the growing interest on the subject.

The second and third weeks of the workshop were organized as focus weeks with about 100 participants. Each day was devoted to a particular topic, and it involved from four to six seminars followed by extensive discussions. The first seminar of the day provided an introduction on the state of the art, and was usually delivered by one of the workshop organizers. Subsequent talks presented recent results to the large audience. All seminars were done at the blackboard for being pedagogical and fostering discussions. This format of presentation and discussion was particularly useful: the beginners in the field could grasp something of the ongoing research, while the experts could collectively discuss the open issues.

The rest of the workshop followed the standard GGI schedule of informal discussions and one seminar per day. A significant aspect was the large participation: in total over 150 physicists attended, including more than 30 PhD students and young researchers. Significant progress was made on several of the topics described earlier. Some results have already been published and many others will appear in the coming months. The larger size of the meeting with respect to earlier events was very effective in permitting the participation of a broader theoretical physics community, thus allowing the dissemination of the new methods and results.

More specifically, the workshop played a significant role in the advancement of the following topics:

- the study of the bootstrap for conformal field theories with boundaries;
- the structure of conformal partial amplitudes;
- the RG flows in theories with dimension $4<d<6$;
- the exact results in supersymmetric conformal theories;
- the understanding of the web of dualities among three-dimensional theories;
- the relation between the conformal bootstrap and the S-matrix bootstrap.

Acknowledgments

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