Il Colle di Galileo

Advances in Non-equilibrium Statistical Mechanics: large deviations and long-range correlations, extreme value statistics, anomalous transport and long-range interactions

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Abstract. The workshop brought together leading researchers, young scientists and PhD students working in different areas of non-equilibrium statistical mechanics. The recent progress made in various fields of this broad discipline made the workshop timely and successful. Out-of-equilibrium systems are typically characterized by generic long-range correlations resulting in a variety of collective phenomena which are still awaiting an overall theoretical understanding. The main topics covered were: large deviations, current fluctuations, fluctuation relations, anomalous transport, statistics of extreme events and slow relaxation in non-equilibrium processes. The relation with equilibrium systems with long-range interactions, where ensembles become inequivalent, was also addressed. This workshop has strengthened the interaction among different communities and has led to the exploration of open problems and new research directions. Joint research in this field, combining theoretical investigations with an empirical component, has led to particularly fruitful exchanges.

Keywords. Large deviations, fluctuation theorems, extreme events, simple exclusion processes, long-range correlations, long-range interactions

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Introduction

When a system is maintained out of equilibrium by the application of an external driving field, e.g. a bar of metal placed in contact with two heat-baths at different temperatures $\mathbf{T_a} > \mathbf{T_b}$, a current flows into the system, a heat-current in the case of the temperature field (see Fig. 1). Although this is a well-established experimental fact, the derivation of the macroscopic properties of the current from realistic microscopic models remains an open issue. This difficulty is related to the question of extending statistical mechanics to the non-equilibrium realm.

Figure 1. Heat-current between two thermal baths at different temperatures.

Considerable progress in our understanding of non-equilibrium systems has been made over the last two decades. This progress has been achieved through a detailed analysis of simple toy models such as the asymmetric simple exclusion process (ASEP), the zero-range process (ZRP), the Katz-Lebowitz-Spohn (KLS) model and others.

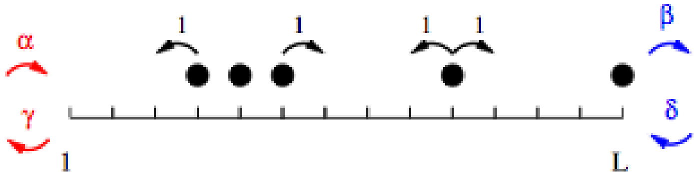

Figure 2. Symmetric simple exclusion process.

An example of such a model, the symmetric simple exclusion process (SSEP), is illustrated in Fig. 2. Particles can hop left and right with equal probability from a given site of a lattice, provided the site to which they jump is not occupied. Reservoirs inject and extract particles with rates α and γ from the left end and δ and β from the right. A density current is established in the system, in complete analogy with the heat-current of the previous example. This model can be considered as a microscopic realization of the previous macroscopic phenomenon. The advantage of having such a model is that one can ask specific questions and obtain detailed analytical results. It emerges that several properties of the current probability distribution have a general and universal validity, irrespective of the considered model. A plethora of different rigorous methods have emerged which make it possible to obtain a detailed knowledge of the many-body invariant measure at microscopic level. On the other hand, a general theory has been formulated for diffusive systems, the so-called macroscopic fluctuation theory (MFT), which gives a universal formula for the joint probability distribution of the spatial and temporal dependence of the density and the current, in analogy with Einstein's formula that relates fluctuations in equilibrium to entropy. The discussion of these topics was the central theme of the workshop, conference and tutorials.

Large deviations and extreme events

The current is a fluctuating quantity, the probability that it flows opposite to the drive is non zero. Some general fluctuation theorems (Gallavotti-Cohen, Jarzinsky-Crooks, Harada-Sasa) have been derived, which yield information on fluctuations in driven systems. Using large deviation theory steady state probability distribution functions may be analysed. The large deviation function plays the role of free energy density in equilibrium systems. However, while in equilibrium the free energy density obeys certain rules (such as being analytic except at the phase transition point, convexity and others), no such general rules are known for large deviation functions of non-equilibrium systems. It is well known that non-equilibrium stationary distributions generically exhibit long-range correlations, even if the dynamics governing the system is local as in the SSEP of Fig. 2. Moreover, it has been found that the large deviation function is non-analytic in the general space of density profiles of the fluctuating variables. Remarkably, it is possible to obtain detailed insight, not only into stationary correlations, but also into the space-time structure underlying extreme events, such as density profiles which produce rare anomalously large currents of mass or energy in systems driven by boundary gradients or bulk fields. Closely-related exact results include the derivation of universal current distributions in driven diffusive systems and novel universal distributions for extreme values of correlated random variables. From a rather different point of view, recent studies of equilibrium systems with long-range interactions have shown that these systems share some common properties with driven non-equilibrium systems such as ensemble inequivalence. With these results at hand, during the workshop it was possible to address new problems and explore the general validity of some of these findings. For example, the knowledge of space-time realizations underlying extreme events provides clues to the dynamical mechanisms that cause extreme events and hence how they can be triggered or suppressed. Exploring correspondence between driven systems and equilibrium systems with long-range interaction has yielded useful insights into the steady state and dynamical properties of driven systems.

These results may have an impact in an interdisciplinary perspective. Indeed, extreme events in nature such as earthquakes, floods, droughts etc. are rare, but have serious and often devastating consequences when they occur. Understanding the probability of occurrence of such events and their associated statistics, popularly called Extreme Value statistics (EVS), is thus of paramount interest and has been studied extensively by engineers, mathematicians and statisticians over the last century. In EVS, one is typically concerned with the distribution of the maximum (or minimum) of a set of random variables. The subject is clearly understood when the underlying random variables are uncorrelated or weakly correlated, with short-range interactions between them. However, our knowledge of EVS is still limited when the random variables are *strongly* correlated, as is typically the case when there are *long-range* interactions between these random variables. It has now become progressively clear that EVS is also highly relevant in many physical systems, notably in disordered systems such as spin glasses, polymers in disordered medium and in a variety of constrained optimization problems in statistical physics. In such systems, the low-temperature properties are typically governed by the ground state, i.e., the configuration with the *minimum* energy, and thus EVS is clearly relevant there. However, in most physical systems the relevant random variables (such as the energies of different polymer paths in a fixed quench disordered environment), of which the extreme is being sought, are typically *strongly* correlated. Over the last few years, there has been some theoretical and experimental progress in understanding the EVS of strongly correlated variables. One particular success story is the computation of the distribution of the maximum eigenvalue of a Gaussian random matrix, the famous Tracy-Widom distribution. The eigenvalues of a random matrix can be described as a charged gas with *long-range* Coulomb interaction between them. Subsequently it turned out that the Tracy-Widom distribution is ubiquitous and has been found in many systems, such as: in directed polymers in (1+1)-dimensions; in the height distribution of a fluctuating *non-equilibrium* interface belonging to the famous Kardar-Parisi-Zhang universality classes; in sequence-matching problems in biology; in the distribution of the conductance of a quantum dot, and in many other systems. Several recent experiments, in liquid crystals and in optical fibre lasers, have measured this distribution. The issue of *large deviations*, that describe extremely large *atypical* fluctuations of the extreme observables, thus going beyond the Tracy-Widom distribution, are also of much current interest and recent sophisticated experiments in fibre lasers have been able to measure such tiny probabilities. Thus, the EVS of strongly correlated random variables is a subject of great interest and currently intense activity in both the statistical physics and the probability theory communities. It has important connections with disordered systems, non-equilibrium systems and long-range systems on the one hand, and with random matrices and the probability theory of large deviations on the other hand. During the workshop some progress was made in the study of extreme value statistics of particular models of strongly correlated systems, and there is now hope for the emergence of a general theory. From the experimental point of view, one expects that the situation will improve since the databases will grow. However, here special attention has to be paid to the problem of handling the outliers. An important aspect of handling the outliers is theoretical. Namely, how to build a stochastic model and compare it with extreme events in experiments, if one is not entirely sure what drives the fluctuations at large deviations from the average. The usual procedure of including equilibrium-type noise in the description (additive or multiplicative) becomes questionable if it is recognized that e.g., a climatic process on the timescale of many years is driven by noise which strongly violates time-reversal invariance. A topic discussed in the workshop was how to develop and understand models with non-equilibrium noise. There are, of course, many possibilities for considering more definite, particular problems which are beautiful in themselves (the effect of extreme events in reaction-diffusion problems, the maximum velocity of crack- or avalanche propagation, the description of extremes in multifractals, etc.). While the first steps have been made in these directions, clearly there is much still to be done. At the workshop, researchers with expertise in various subfields exchanged ideas in order to develop a systematic approach towards understanding and classifying the EVS of strongly correlated variables.

Anomalous transport and thermodynamics in low dimensional systems

This research area has grown significantly in the last decades and has involved an increasing number of research groups all over the world. It should be pointed out that experimental studies in nanophysics, material science and nonlinear optics have benefitted significantly from the many hints emerging from theoretical investigations. For instance, one could mention the experimental verification of anomalous heat transport in carbon nanotubes as a specimen of quasi-1d systems, or the formation of localized breather-like excitation in coupled optical waveguides. In a more general perspective, it should be mentioned that anomalous transport properties have been detected in several kinetic and dynamical models in one and two spatial dimensions. It has been found that in 1-D systems heat conductivity exhibits a power-law divergence with the system size and it has been conjectured, on the basis of several numerical simulations and analytic estimates, that the power-law may characterize different universality classes associated with the basic symmetries of the model in question that are preserved within the hydrodynamic limit. In particular, the case of zero pressure models (i.e., models that are defined in terms of the left-right symmetry of the interaction) is expected to exhibit a different scaling with respect to asymmetric models (e.g., models dominated by a leading odd nonlinearity in the interaction). Anyway, in all these cases the anomalous behaviour can be traced back to an effective description of the mode dynamics by a generalized fractional Brownian motion. Despite the many achievements obtained in this field, problems concerning universality are still challenged by contradictory numerical results. In particular, in the case of asymmetric interactions, normal conductivity seems to be restored when the amplitude of the leading odd nonlinearity is sufficiently large with respect to other regularizing terms. On the other hand, it is still a debated question which kind of mechanisms are at work in observing an effective normal conductivity in very large systems which are expected to approach a power-law divergence of the thermal conductivity for longer chains. At the workshop a new theoretical hydrodynamic approach was presented, which might lead in the near future to the solution of many of these puzzles. Another interesting aspect of low-dimensional systems is the possibility of exploring situations where their thermodynamic description admits the existence of negative heat capacities and temperatures. Such anomalous thermodynamic properties have been observed in models of non-extensive systems (e.g., systems with long-range interactions) or characterized by the spontaneous formation of topological excitations in non-equilibrium metastable states, as has been observed in the so-called Discrete Nonlinear Schroedinger (DNLS) equation. A satisfactory understanding of this scenario was addressed during the workshop and a coherent effort in this direction was made concerning possible applications in plasma physics as well as in quantum optics and Bose-Enstein condensates. A coherent theoretical framework for all these phenomena is still lacking. However, the workshop constituted a serious attempt to explore relations and analogies among the various models and this effort is expected to yield some crucial advances in the future.

Long-range interactions

Long-range interactions are found in astrophysics, plasma physics, hydrodynamics, atomic physics, nuclear physics, solid-state physics and disordered systems. If, for sufficiently large distances, the two-body potential decays with a power smaller or equal to the space dimension, then the interaction is long-range. It is usually stated, and experimentally verified for many physical systems with short-range interactions, that in the thermodynamic limit the predictions of statistical mechanics do not depend on the chosen ensemble. An important feature of long-range systems is that *ensembles can be inequivalent*. Therefore one of the main objectives of the theoretical studies of these systems is to explore the relation between the various ensembles, probe the conditions under which they become inequivalent and determine the relevant ensemble appropriate for a given experimental setup. Ensemble inequivalence is not merely a mathematical drawback, it is a consequence of the fundamental physical properties of long-range systems. One of the most striking features of long-range systems is the possibility to display *negative specific heat* in the microcanonical ensemble, which is a consequence of ensemble inequivalence. As far as nonequilibrium properties are concerned, long-range systems demonstrate peculiar behaviour. The approach to equilibrium can be extremely slow: it takes place on time scales that increase with system size. The state of the system during this long transient is *quasi-stationary*, since its very slow time evolution makes it possible to define slowly varying macroscopic observables, as in the case of local equilibrium or quasi-static transformations. The explanation of the widespread presence of quasi-stationary states relies on the study of the dynamical properties of systems with long-range interactions. In the continuum limit, long-range interacting systems admit a kinetic description in terms of the *Vlasov equation*. Within this scenario, quasi-stationary states correspond to stable stationary states of the Vlasov equation. The finite size system relaxes to equilibrium due to corrections of the Vlasov equation, the study of which must be performed in the context of appropriately devised *kinetic theories*. The problem of developing a rigorous statistical mechanics treatment of quasi-stationary states was dealt with during the workshop, producing a lively debate on the different theoretical proposals. A system can be driven out of equilibrium by subjecting it to the action of an external force or an electric field, or coupled to thermostats at different temperatures. Even in the case in which the bulk dynamics is conserving and local (as in the examples in Figs. 1 and 2), such systems are characterized by long-range correlations, which lead, e.g., to phase transitions and long-range order even in one dimension. In some models, features characteristic of long-range interactions, such as ensemble inequivalence, have been found. In specific cases it can even be shown that, in a particular range of parameters where detailed balance is satisfied, the weight of configurations is given by an effective Hamiltonian which has explicit long-range terms. This research direction was further explored during the workshop, and the connection between *long-range correlations* and long-range interactions was further clarified. It has become progressively clear that the ubiquitous presence of long-range forces calls for an approach that integrates different methodologies. Questions still open in this field are: obtaining necessary and sufficient criteria for ensemble inequivalence; exploring the conditions for the experimental verification of the effects induced by ensemble inequivalence, such as negative specific heat; determining the conditions for the evolution towards quasi-stationary states and characterizing the law of divergence of the timescale with system size; checking the stability of quasi-stationary states under the effect of external perturbations; developing appropriate kinetic theories for systems with long-range interactions; and, unveiling the possibility of describing long-range correlations in driven systems in terms of long-range interactions. Several aspects related to these topics were discussed during the workshop.

Some data on the workshop

Almost 150 researchers participated in the workshop activities, including several PhD students and post-doctoral fellows, who had the chance of giving either an oral or a poster presentation at the conference. During the workshop, a training week was organised in which several local PhD students and post-docs participated. The lectures given during this week resulted in some cases in lecture notes that are available on the GGI website and on the arXiv. Within the wider context of the workshop, a one-week conference sponsored by IUPAP took place. About 70 participants from many different countries were present at the conference and 48 gave invited talks.

The workshop was perceived by the Italian community working in statistical mechanics as a unique opportunity to meet and discuss with the best experts in the field. Many colleagues and students visited GGI during the workshop at their own expense. The organisers of the Parma meeting on “Statistical mechanics and complex systems” (June 25-27) took advantage of the presence of so many renowned scientists in Florence to invite some of them.

In terms of output, already about 40 preprints acknowledging GGI, which were directly generated by the workshop, are available on the GGI website and on arXiv (many others are expected to arrive). A special issue of a leading journal in the field, the *Journal of Statistical Mechanics: Theory and Experiment* (IOP), titled “Correlations, fluctuations, rare events and transport in non-equilibrium systems” has been scheduled.