



Il Colle di  
Galileo

# Geometry of Strings and Fields

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Francesco Bonechi (INFN Sezione di Firenze), Alberto Cattaneo (Zurich University), Sergei Gukov (Caltech University), Martin Rocek (Stony Brook University), Domenico Seminara (Università di Firenze), Maxim Zabzine (Uppsala University)

**Abstract.** This workshop focused on the interactions between string theory and geometry. Since its birth, the development of string theory has triggered major progress in mathematics. Experts from both communities from all over the world took part and created an environment which favored a fruitful scientific exchange. Among the key topics discussed during the workshop, we should mention derived algebraic geometry and topological field theory, localization formulas in supersymmetric quantum field theories, links between conformal field theory, susy and integrability, new trends in knot theory.

**Keywords.** String theory, geometry, topological quantum field theory, localization, derived algebraic geometry

## 1 Scientific Motivations

This program was devoted to geometrical issues motivated by string theory and to recent developments in string theory and related physical fields that are of great mathematical interest.

Ever since the birth of string theory, interaction with geometry has been one of the primary driving forces that has led to progress in superstring theory. On the one hand, string theory has generated many new geometrical concepts; on the other hand new ideas from geometry have often found their first applications in string theory. These topics include vertex algebras, conformal field theory, mirror symmetry, topological field theory and string theory, exact solutions of supersymmetric gauge theory and noncommutative field theory. Exciting recent developments include the matrix model approach to  $N=1$  gauge theory, open string mirror symmetry, the derived category approach to D-branes on Calabi-Yau manifolds, geometric transitions, proof of the  $N=2$  Seiberg-Witten solution by instanton methods, wall crossing formulas, the relation between Langlands program and supersymmetric gauge theories, indications of integrable structures in super Yang-Mills theory and AdS string theory.

## 2 The Program

The program lasted eight weeks, including one week for the conference and the two focus weeks. Around one hundred researchers participated, a quarter coming from Italian institutions and the rest from all over the world; eighteen were PhD students who were mainly present during the two focus weeks and the conference. During the two months we had a balanced participation of both theoretical physicists and mathematicians.

The workshop was organized in such a way as to give participants the opportunity for informal discussion while also giving the speakers the opportunity to give a detailed presentation of their work. Therefore, a typical day was organized with a two-hour seminar in the morning and the afternoon left free for informal discussion. Occasionally a seminar was organized in the afternoon too.

The focus weeks featured a more intense program comprising a series of lectures aimed at presenting both advanced topics and more standard material conceived mainly for students. The first focus week took place from the 2<sup>nd</sup> to the 7<sup>th</sup> of September and hosted five series of lectures:

- D. Calaque, “*Derived symplectic geometry and topological field theories*”;
- R. Heluani, “*(Sheaves of) vertex algebras and stringy manifold invariants*”;
- V. Pestun, “*Localization of supersymmetric quantum field theories on curved background*”;
- N. Rozenblyum, “*Branes from a derived point of view*”;
- R. Von Unge, “*The secret life of sigma super-models*”.

The second focus week was organized from the 7<sup>th</sup> to the 12<sup>th</sup> of October and included five courses:

- M. Aldi, “*Hamiltonian Quantization of Loop Spaces and Vertex Algebras*”;
- G. Ginot, “*An introduction to differentiable stacks and moduli space*”;
- P. Mnev, “*Around moduli spaces of flat connections*”;
- J. Qiu, “*Localization of 5D super Yang-Mills*”;
- E. Witten, “*Topics on Superstring Perturbation Theory, Chern-Simons theory and Khovanov Homology.*”

The lectures of both focus weeks were filmed and they are now available to students and researchers on a YouTube channel named after the workshop.<sup>1</sup> The conference was held in the week after the first focus week and we had 19 invited speakers. The topics ranged from derived geometry to topological field theory and the relation between geometry and supersymmetric gauge theory.

<sup>1</sup> The html address is: <https://www.youtube.com/channel/UCAHF6d01vBSv3FpAjDfldUQ>

More details on the topics covered can be found on the web-page of the conference.<sup>2</sup> The atmosphere during the conference was friendly and led to many questions and fruitful interaction between mathematicians and physicists.

Among the topics that were discussed during the workshop, particularly hot was the derived algebraic geometry introduced by Vezzosi and Toën. This is a far-reaching extension of ordinary algebraic geometry which performs well with respect to quotient and intersection of subvarieties that recently attracted a lot of attention in the world of algebraic geometry. Their recent work on shifted symplectic structures, discussed during the workshop in the lectures by Calaque and in the talk by Vezzosi during the conference, attracted attention and stimulated much discussion. In fact it was immediately clear to people with a background in Topological Field Theory that this construction was intimately related to the AKSZ construction of the solutions of the master equation in the Batalin-Vilkovisky method for the quantization of gauge theories. At the same time, the two approaches are technically quite different, one existing in the smooth category and the other in the algebraic: the relation must be put down in writing and its clarification promises to give a new and original point of view to this basic construction of TFT. Getzler's talk during the conference can be seen as a first step in this direction.

Recent years have seen increasing interest in constructing and studying supersymmetric gauge theories on curved manifolds. The supersymmetry algebras of these theories are deformed to accommodate background curvature and the resulting partition functions can be computed via a particular saddle-point procedure, known as the supersymmetric localization technique. Thanks to this technique, an impressive number of new exact results have been derived for supersymmetric theories in different dimensions, mainly when formulated on spheres or products thereof. These exact results in turn stimulated further remarkable discoveries such as AGT-type correspondences, tests of non-trivial large  $N$  scalings, the  $Z$ -minimization and  $F$ -theorem and a new method, alternative to mirror symmetry, for the computations of Gromov-Witten invariants. During the program we had many contributions centered on this subject and the related areas. For instance, G. Festuccia gave two introductory talks on a uniform treatment of rigid supersymmetric field theories in a curved space-time  $M$ , focusing on four-dimensional and three-dimensional theories with different number of supercharges. The general results on the dependence of the partition function of the theory on the geometry of  $M$  were particularly significant. J. Qiu in his lectures during the second focus week gave an exhaustive presentation of last year's progress in the problem of evaluating the partition function of supersymmetric gauge theories in five dimensions. He also discussed their large  $N$ -limit in connection with the possibility of using 5D MSYM to define the  $(2, 0)$  theories in six dimensions.

<sup>2</sup> A detailed schedule of the workshop and of the conference can be found at [http://theory.fi.infn.it/seminara/Geometry\\_of\\_Strings\\_and\\_Fields/Main.html](http://theory.fi.infn.it/seminara/Geometry_of_Strings_and_Fields/Main.html).

The links between supersymmetric gauge theories, integrability and conformal field theories (CFT) in two dimensions were instead explored by N. Nekrasov and S. Shatashvili in their talks. The former discussed in detail the correspondence between correlators of chiral observables in 4D supersymmetric theories and holomorphic blocks of some CFT in two dimensions [AGT correspondence]. He also suggested that this connection might be understood in terms of some novel non-local symmetry between theories in  $D = 2$  and 4. The latter reviewed how the problem of determining supersymmetric vacua in theories with four supercharges in various dimensions is connected to Bethe eigenstates of lattice models of statistical physics as well as to the quantized Hitchin system.

Finally we should mention the contribution of E. Witten who gave an overview of his recent activity attracting the attention of both physicists and mathematicians. His first talk was devoted to discussing the moduli space of super-Riemann surfaces as it appears in the perturbative expansion of superstring theory. His second lecture addressed the problem of the analytic continuation of CS partition function in the coupling constant  $k$ . This allowed him to present a new approach to Jones polynomials starting from his attempt to find a field theory origin for the “volume conjecture”. Finally in his last lecture he described a new approach to Khovanov homology, that is categorification of the Jones polynomials, based on counting the solutions of certain elliptic partial differential equations in four and five dimensions. This topic was also discussed by A. Morozov starting from solutions of the quantum Yang-Baxter equation.

The program also covered other interesting subjects such as flux and non-geometrical backgrounds, instantons in string and field theory, large  $N$  and topological strings. More details on the daily schedule, slides of the talks and links to the videos of the lectures can be found on the webpage of the program.