Il Colle di Galileo

Randomness, Integrability and Universality

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Abstract. In spring 2022, the Galileo Galilei Institute for Theoretical Physics hosted a seven-week Workshop on *'Randomness, Integrability and Universality'*. The Workshop addressed a series of questions on exactly solvable models of statistical mechanics, having numerous ties and overlaps with various problems in random matrix models, probability, representation theory, and combinatorics. Much recent progress in these areas exploits the underlying notion of quantum integrability. Here we report on the scientific motivations and background of this activity and on its main outputs.

Keywords: statistical mechanics, lattice models, quantum integrability, exact results, combinatorics, tilings, dimers, growth processes, limit shape phenomena, random matrices, random surfaces, determinantal processes, discrete holomorphicity.

Scientific background

The last two decades have seen an increasing interplay between theoretical physics, probability theory, and combinatorics, particularly in the study and application of exactly solvable models of statistical mechanics, with several remarkable developments.

Examples of such constructive interaction include significant progress in understanding the structure of random surfaces and limit shape phenomena; their relationship to transport phenomena in inhomogeneous quantum quenches, and with stochastic growth processes. Other examples include the discovery of integrability in gauge fields and strings; the application of discrete holomorphicity in dimer models and the emergence of 'integrable probability' as an area at the interface of probability theory and integrable models in statistical mechanics. In addition to all this, we should mention the considerable progress made at the interface of statistical mechanics, probability and integrability, which resulted in the rigorous characterization of the Kardar-Parisi-Zhang universality class. Such

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models of statistical mechanics, like dimer models and the six-vertex model, were instrumental in these developments.

Let us discuss in further detail some of the above mentioned topics:

- *Random tilings, random surfaces, and limit shape phenomena.* Random tilings in two dimensions provide an important class of discrete geometric models that exhibit critical behavior. Their large distance critical behavior is expected, and in some cases proved, to be described by conformal field theories. For example, random tilings by lozenges have been analyzed in great detail (together with their `arctic curves', limit shapes, and spatial phase separation phenomena emerging in the scaling limit). This analysis is based on the free-fermionic nature of the problem and its equivalence to dimer models. It heavily involves combinatorics, the theory of random walks, and random matrix models. On the other hand, despite some significant recent progress, not much is known concerning more complex random tiling models that go beyond the class of free-fermion models, but are still Bethe Ansatz integrable, such as the six-vertex model, or square-triangle tilings. Further understanding of these models, with the determination of their limit shapes, and more generally the full characterization of their behaviour in the scaling limit, would be highly desirable.
- *Limit shapes and inhomogeneous quantum quenches.* `Quantum quench' in strongly interacting integrable fermionic models is a paradigmatic protocol in the study of out-of-equilibrium quantum dynamics. Choosing an inhomogeneous initial state, `light-cone' effects and anomalous transport properties are observed. In the case of the quantum XXZ chain, if the quantum evolution is performed in discrete imaginary time, the light cone and the magnetization appear to be in exact correspondence with the arctic curve, and the limit shape of the corresponding six-vertex model, respectively. The consequences of this remarkable property are still to be explored, and should lead to significant advances on both classes of models.
- *Khardar-Parisi-Zhang in non-equilibrium and random systems.* A central challenge in statistical physics is to describe non-equilibrium systems driven by randomness, such as a randomly growing interface. Over time and after zooming out, such an interface will generally approach a deterministic limit shape. Random fluctuations around this limit shape are believed to be universal in scale and statistical description, depending on the growth mechanism and randomness only via simple scaling parameters (KPZ universality). 'Exactly solvable' examples provide the most complete access to this universal behavior and allow the testing of the universality hypothesis on a variety of systems. In recent years, remarkable achievements are the exact solution of the KPZ equation and its striking experimental confirmation in turbulent liquid crystals. Currently, the subject is actively developing with outstanding progress in understanding the nature of stochastic processes of

the Macdonald type, stochastic differential equations, random matrices, directed polymers and interacting particle systems. The KPZ phenomenon in non-equilibrium transport and quantum quenches is a very important rapidly developing subject.

- *Integrability in gauge and string theory.* Major progress in the study of gauge-string dualities came recently from applications of integrability to higher dimensional supersymmetric gauge theories and string theory. A flurry of new developments concerns computations of correlation functions, Wilson loops and gluon amplitudes, and the fallout might be useful for less supersymmetric and more realistic theories like the QCD. There is a close relationship between these developments and those pertaining to other applications of integrability, such as the study of quenches, non-equilibrium physics or correlation functions in one-dimensional quantum systems. A particular focus will be placed on developing the common mathematical tools necessary to tackle these problems. The connection between the space of vacua of N = 2 supersymmetric gauge theories and the integrable structure of two dimensional conformal field theories is also a constant source of progress in our understanding of field and string theory. The link between limit shape phenomena and topological strings is also of particular interest.

Essential components of many recent developments in the above mentioned areas are, on the one hand, the interplay between discrete and continuous formalism, and, on the other hand, integrability, make it possible to calculate exact results.

Recent developments in the above mentioned topics are not only highly important in their own right, they also carry deep implications across many different fields ranging from traditional models of statistical mechanics, disordered systems, classical and quantum non-equilibrium phenomena, to algebraic geometry, gauge and string theory, quantum gravity and random matrices.

The Workshop

The Workshop followed similar programs on the subject, held over the last decade at the Newton Institute (Cambridge), Schrödinger Institut (Vienna), CRM (Montreal), Institut Henri Poincaré (Paris), MSRI (Berkeley), Kavli Institute (Santa Barbara) and the Simons Center (Stony Brook).

The aim of the Workshop were to bring together experts in conformal field theory, integrable systems, analysis, probability theory, combinatorics and representation theory, for the purpose of:

i) representing the state of the art in a number of currently active areas of research into statistical mechanics, and related to integrability, random geometry and combinatorics, as described above;

ii) providing an opportunity for further cross-fertilization between these different areas, stimulating new ideas, and further advances;
iii) giving young researchers the opportunity to learn about and be encouraged to contribute to the exciting new developments in these areas.

The main topics of the Workshop included:
- Random tilings and limit shape phenomena;
- Random matrices, determinantal processes and KPZ universality class;
- Quantum integrability and correlation functions;
- Lattice models and combinatorics;
- Integrability in gauge and string theories.

Attendance and funding

This seven-week workshop took place from April 12 to June 3, 2022. It was essentially the first 'in-person' activity at GGI after over two years of pandemic (with the exception of a couple of shorter events in Fall 2021). In particular, it is worth recalling that in-person activities at GGI were explicitly forbidden in December 2021, "until further notice". The ban was lifted only at the end of March 2022, although keeping a restriction to 40 in-person participants at a time.

With these premises, the organization of the program was indeed a real challenge. First, it was rather difficult to set up such a big event without being sure, until the last two weeks, whether it would even actually take place. Second, the unknowns concerning the pandemic, and therefore the possibility of travelling, prevented potential participants from finalizing their plans until the very last minute. Finally, the strict limitation to 40 participants prevented us from organizing a conference, or a training or focus week.

Hampered by these issues, and having to cope with a huge number of applications, the organizers agreed to maximize the number of participants, with the idea of having an average of 35 participants throughout the whole program, a kind of compromise between a standard GGI workshop with about 20 participants at a time, and a conference, with 80 or more attendees.

Despite the above mentioned issues, the program was a considerable success, with over 140 applications, and 108 actual participants (mostly from Europe, due to the residual effects of pandemic), each for an average of two to three weeks.

Most of the Organizers remained for a significantly long time, as follows: Filippo Colomo: seven weeks; Jan de Gier: five weeks; Philippe Di Francesco, Didina Serban, and Herbert Spohn: four weeks each. Unfortunately Nikolai Reshetikhin was unable to visit the program, being in China at the time and subject to the very tight Covid-19 regulations enforced there.

In terms of financial support, 32 participants (for a total of 87 weeks) funded their own participation, six participants (for a total of 18 week) were GGI Simons

Fellows, while 70 participants (for a total of 145 weeks) received support with accommodation expenses from GGI, within the standard workshop funding. The Simons GGI visitors were Pavel Bleher, Kurt Johansson, Ivan Kostov, Pierre Le Doussal, Fedor Smirnov and Paul Wiegmann. The workshop did not receive any financial support from sources other than the GGI and the Simons Foundation.

The number of junior researchers was remarkably high, approximately 50%, also thanks to those additional GGI funds devoted to supporting visits by young scientists. Particular care was given to favour diversity and inclusiveness, with good, but still improvable, results (for instance, 20% of participants were women).

Organization and logistics

Everything went smoothly, thanks to the excellent organization of GGI, and to the great support of GGI staff. Indeed, many participants were impressed by the efficiency of our staff and could not believe that the entire organization relied essentially on four or five people.

We had a few welcome drinks, social dinners, social promenades around Arcetri, which helped create a nice mood among the participants.

Scientific activities and outputs

As said, pandemic-related issues prevented us from having a conference or focus/training week. At the same time, all participants were eager to engage in scientific exchange after two years of limited communication. The chosen format was that of a ‘diluted conference’, with two or three research seminars a day, still leaving sufficient time for discussion and collaboration. The atmosphere was very relaxed and informal, and everybody was free to ask questions, which generated long discussions also after the presentations. This format resulted in a grand total of 90 seminars over the seven weeks of the program.

The main theme of the workshop was that of universality in statistical mechanics, to be tackled rigourously by investigating exactly solvable models. The main purpose was to bringing together experts in low dimensional quantum field theory and statistical mechanics, integrable systems, random matrices, probability theory, and combinatorics, to increase cross-fertilization and boost further advances in the field.

All the intended goals of the workshop have definitely been achieved. Participation records are excellent, and we had extremely positive feedback from many attendees. Interactions between experts from different fields were very lively and constructive. Participation by young researchers and graduate students was also quite significant. Scientific communication during the workshop was enriching and stimulating. Scientific ouput was remarkable, with already dozens of preprints acknowledging the workshop and GGI. Video recording and slides of the talks, available on the Workshop's webpage, provide a valuable source of information on current research in the field. Finally, as a spin-off of the Workshop, a special issue of J. Phys. A, entitled *'Limit Shapes and Fluctuations in Statistical Physics'*, and devoted to some of the main themes of the workshop, is in preparation.

In conclusion, we had a really great workshop! It is still a pleasure for all of us to remember these seven weeks. We are grateful to GGI and INFN for having funded this initiative, to the director and staff of GGI for their priceless support and dedication, and to everyone who attended, for their constructive and collaborative participation. The credit for the success of the workshop goes to all of them.